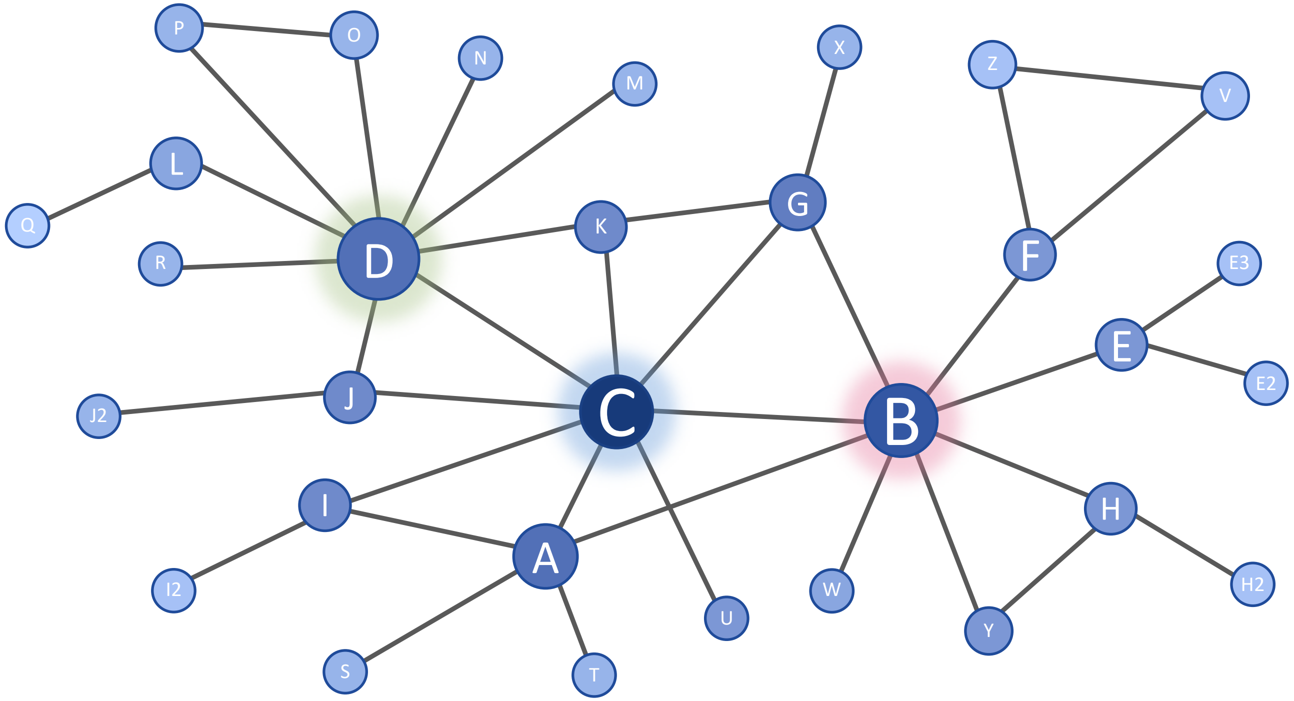
Centrality Measures

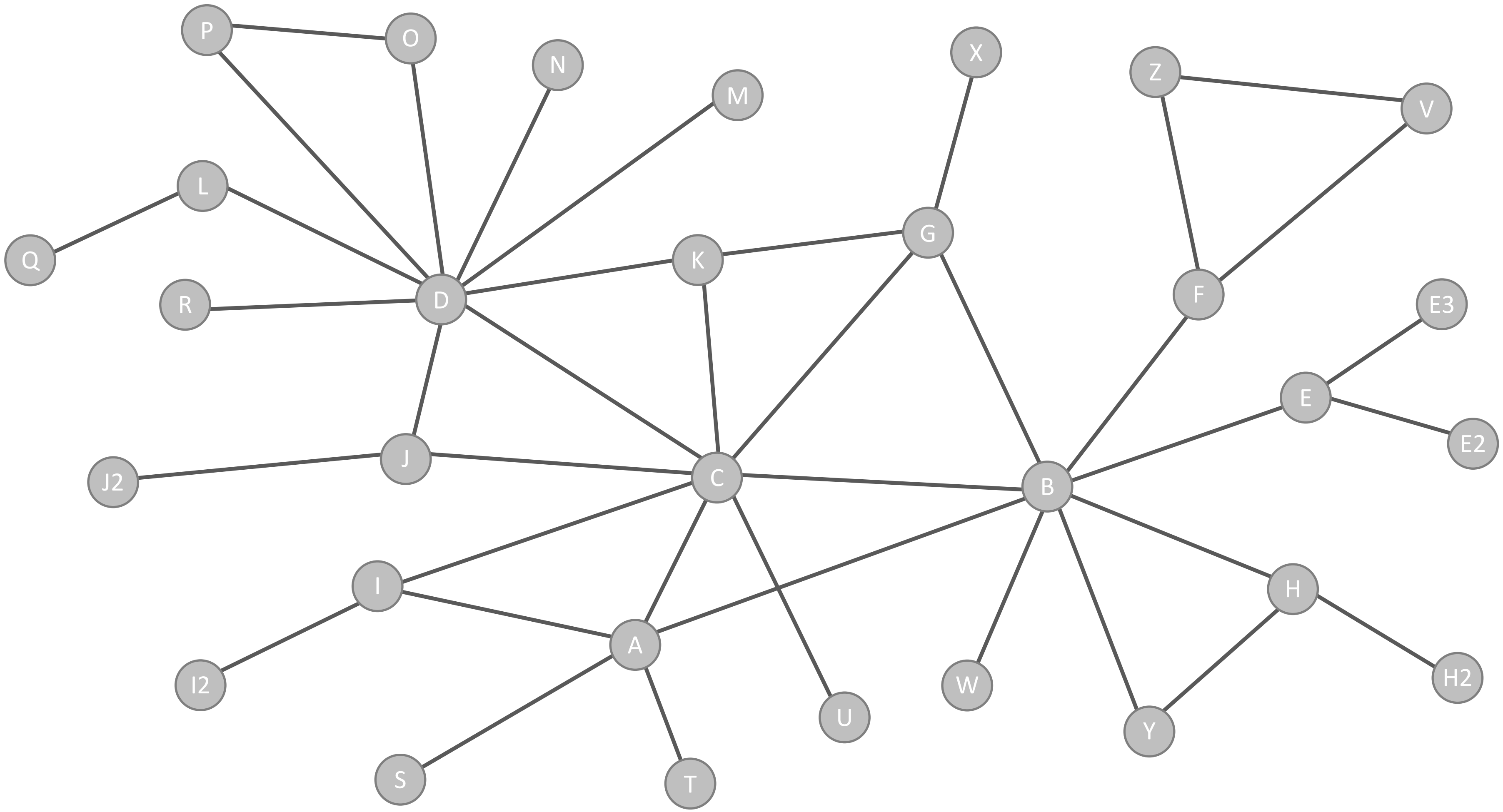


Oskar Skibski

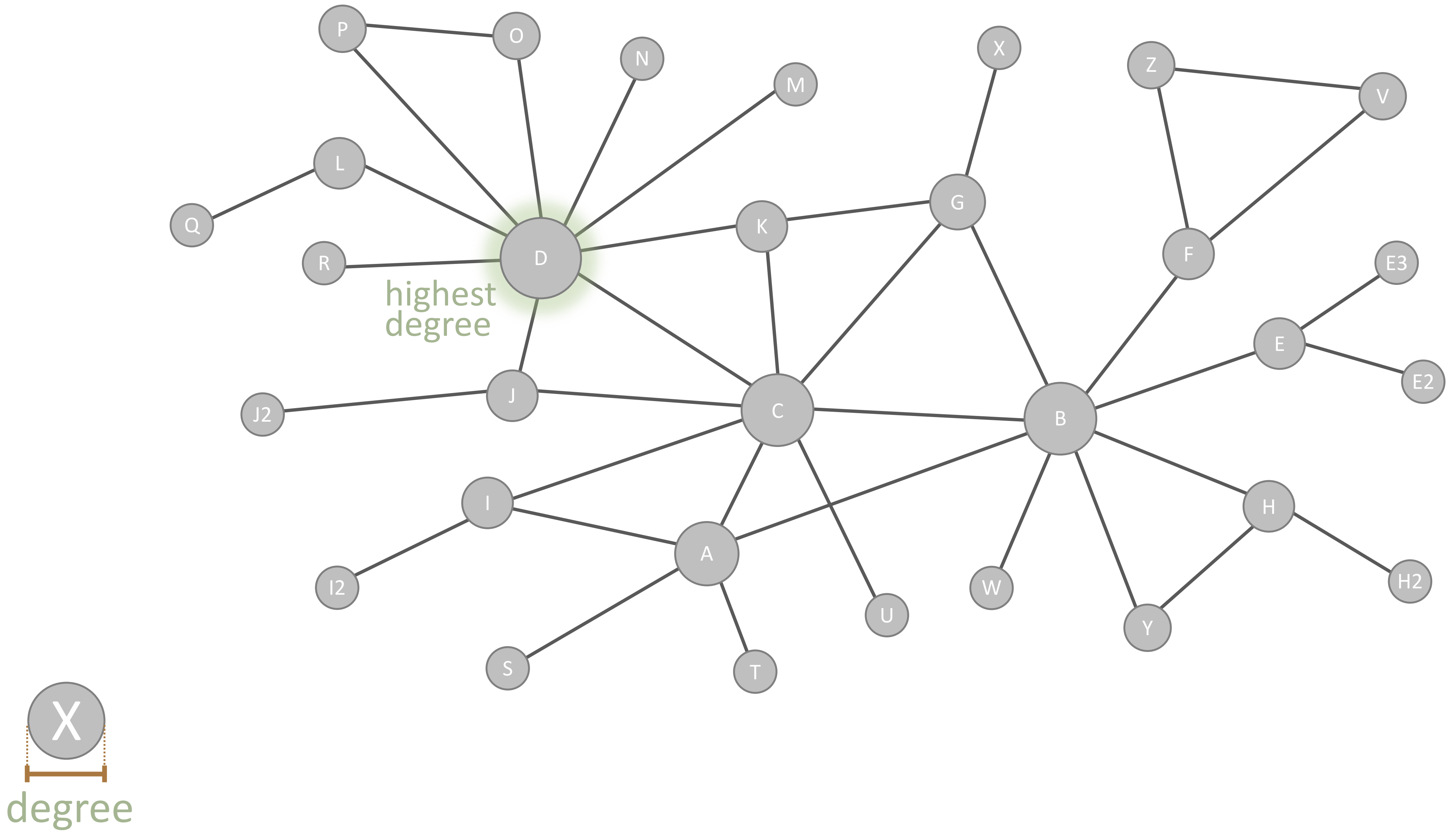
University of Warsaw

Master's seminar, 12.10.2023

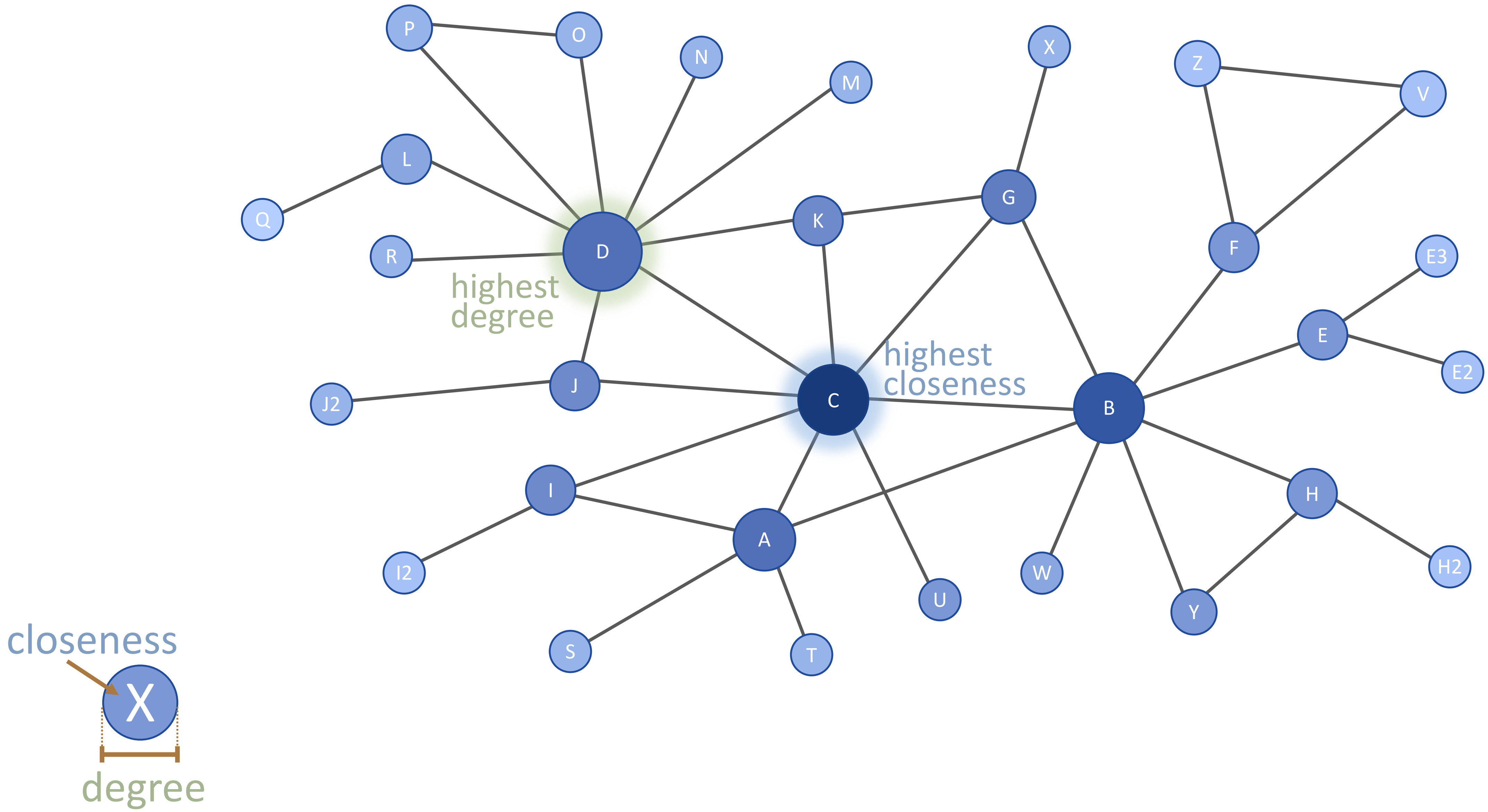
Which node is the most important?



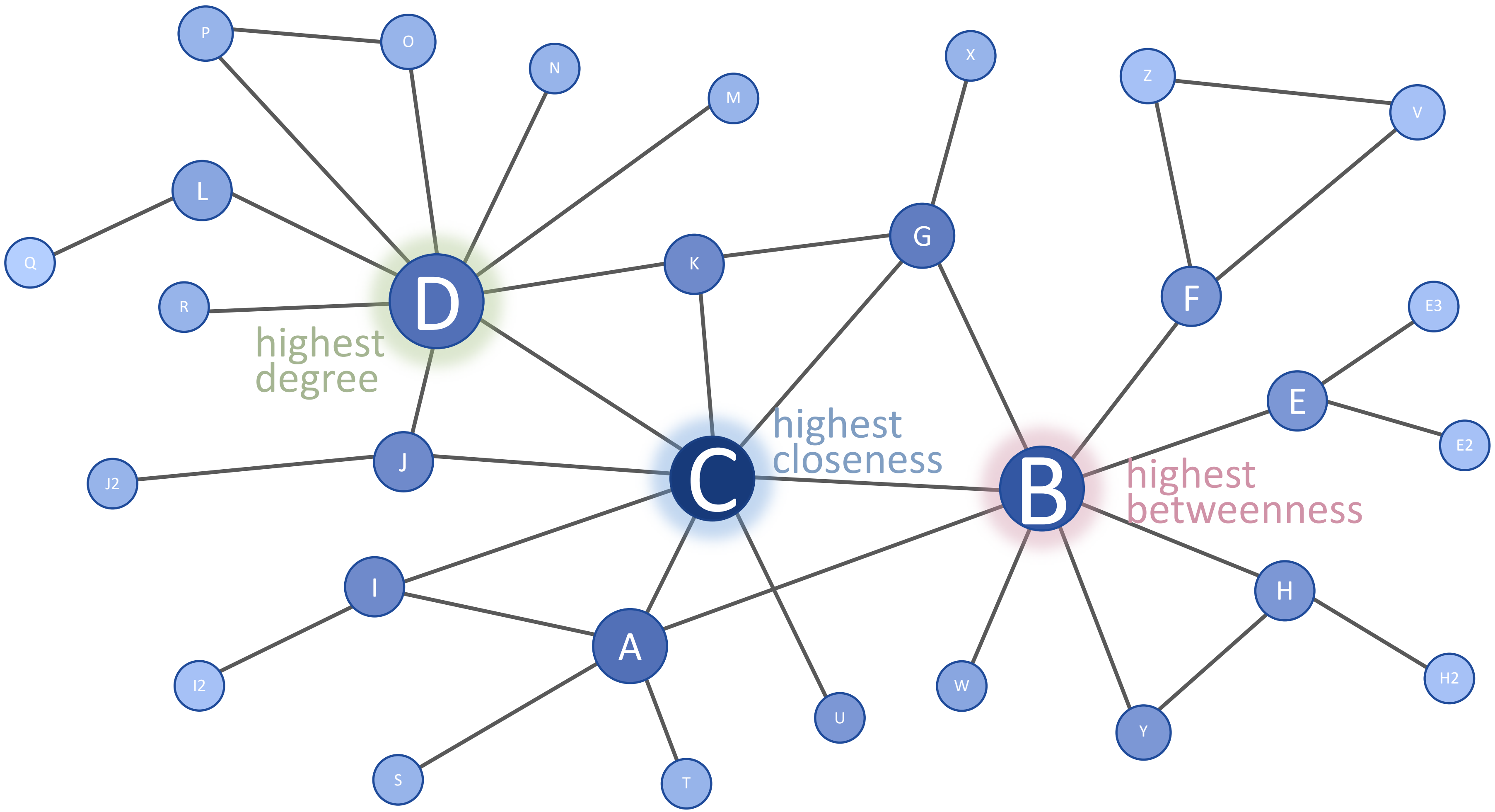
Which node is the most important?

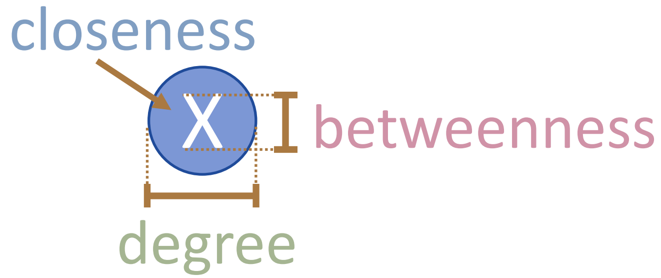


Which node is the most important?



Which node is the most important?





Quantum Hub and Authority (2021)

Signless-laplacian Eigenvector (2021)

Random-Walk Decay (2019)

Stress (1953)

Harmonic (2000)

Local Clustering H-index (2021)

Eccentricity (1995)

Decay (2008)

Random-Walk Betweenness (2003)

Closeness (1950)

Diverse (2021)

Cocktail (2021)

Betweenness (1977)

Random-Walk Closeness (2004)

Degree

Flow Betweenness (1991)

Eigenvector (1987)

All-subgraph (2020)

Katz (1953)

Attachment (2016)

PageRank (1999)

Beta measure (2008)

Connectivity (2001)

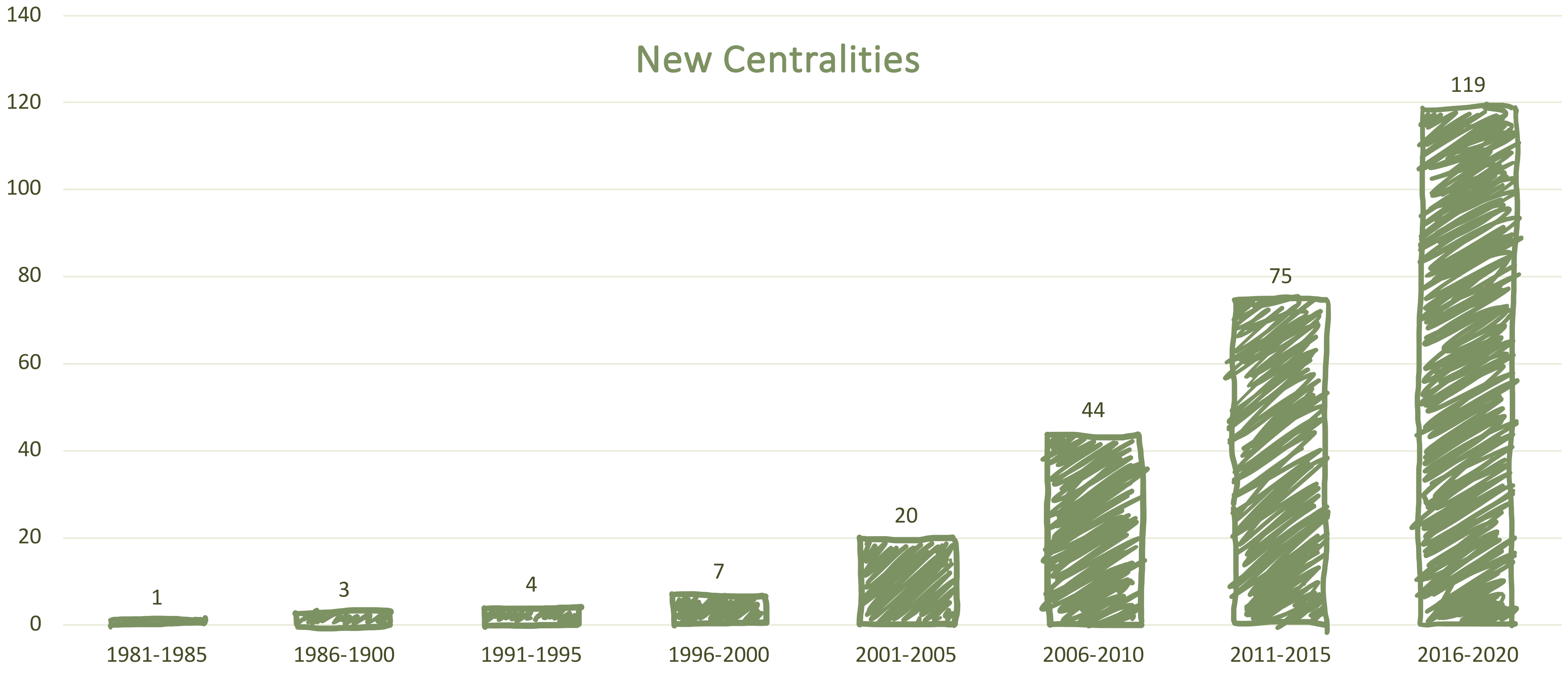
Bonacich (1987)

Seeley Index (1949)

Bonacich Shapley (2021)

Tukey Depth (2021)

Centrality Measures

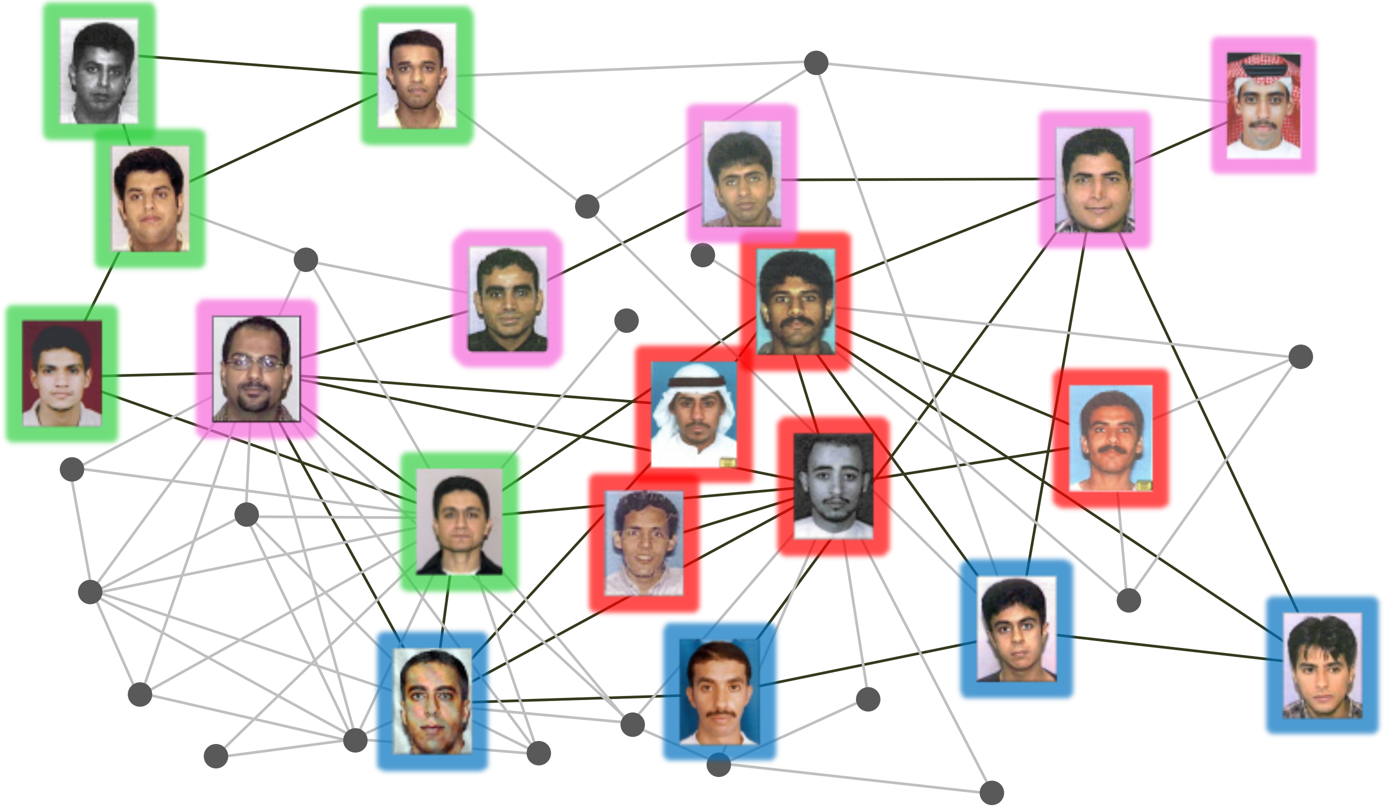


source: http://www.centiserver.org/

Centrality Measures

DEFINITION

A *centrality measure* is a function F that for every graph $G = (V, E)$ and every node $v \in V$ assigns a real value $F_v(G)$.



Applications:

- social networks
- biological networks
- transportation networks
- World Wide Web
- etc.

source: wikipedia

Degree Centrality

Finds nodes with many connections.

$$D_v(G) = |\{u \in V : (u, v) \in V\}|$$

Ignores the whole structure of the network

Distance-based Centralities

Distance-based

Harmonic

Decay

Eccentricity

Closeness

k-Step Reach

Degree

Stress

Betweenness

Flow Betweenness

Katz

Eigenvector

Attachment

PageRank

Beta measure

Connectivity

Bonacich

Katz prestige

Distance-based Centralities

Assess a node by the distances from other nodes in the network.

DEFINITION

A centrality measure is a *distance-based centrality* if there exists a function $f: \mathbb{R}^n \to \mathbb{R}$ such that

$$F_v(G) = f\left(\left(dist_{u,v}(G)\right)_{u \in V}\right)$$

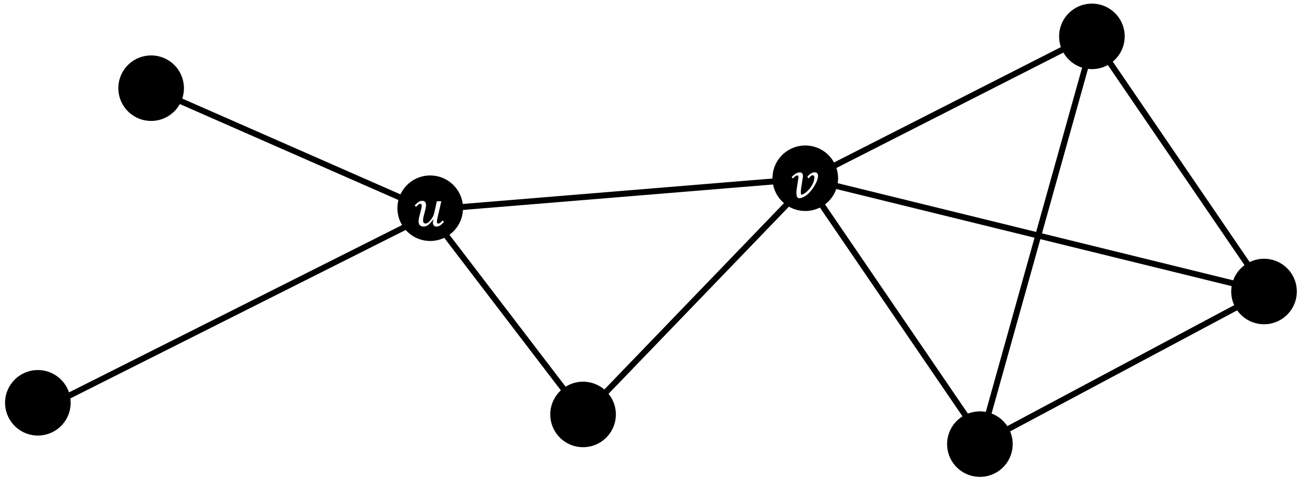
- $dist_{u,v}(G)$ is the distance (length of the shortest path) from u to v
- in directed graphs we usually look at paths to a node

Closeness Centrality [Sabidussi 1966]

Finds the center of a graph.

$$C_v(G) = \frac{1}{\sum_u dist_{u,v}(G)}$$

Note: well defined only for (strongly) connected graphs



Distance-based Centralities

Alternatives:

Harmonic [Rochat 2009]	$H_v(G) = \sum_u 1/dist_{u,v}(G)$
Decay [Jackson 2005]	$Y_v(G) = \sum_u \delta^{dist_{u,v}(G)}$ where $\delta \in (0,1)$

Distance-based Centralities

Alternatives:

Harmonic [Rochat 2009]	$H_v(G) = \sum_u 1/dist_{u,v}(G)$
Decay [Jackson 2005]	$Y_v(G) = \sum_u \delta^{dist_{u,v}(G)}$ where $\delta \in (0,1)$
Degree	$D_v(G) = \lvert\{u \in V \setminus \{v\} : (u,v) \in V\}\rvert$
k-Step Reach	$R_v^k(G) = \left\lvert\left\{u \in V \setminus \{v\} : dist_{u,v}(G) \leq k\right\}\right\rvert$

Distance-based Centralities

Alternatives:

Harmonic [Rochat 2009]	$H_v(G) = \sum_u 1/dist_{u,v}(G)$
Decay [Jackson 2005]	$Y_v(G) = \sum_u \delta^{dist_{u,v}(G)}$ where $\delta \in (0,1)$
Degree	$D_v(G) = \|\{u \in V \setminus \{v\} : (u,v) \in V\}\|$
k-Step Reach	$R_v^k(G) = \left\|\{u \in V \setminus \{v\} : dist_{u,v}(G) \leq k\}\right\|$
Eccentricity	$EC_v(G) = \max_{u \in V}\{dist_{u,v}(G)\}$

Distance-based Centralities

$$\sum_{u \in V \setminus \{v\}} a_{dist_{u,v}(G)}$$

Alternatives:

ADDITIVE (Harmonic, Decay, Degree, k-Step Reach)

Harmonic [Rochat 2009]	$H_v(G) = \sum_u 1/dist_{u,v}(G)$
Decay [Jackson 2005]	$Y_v(G) = \sum_u \delta^{dist_{u,v}(G)}$ where $\delta \in (0,1)$
Degree	$D_v(G) = \lvert\{u \in V \setminus \{v\} : (u,v) \in V\}\rvert$
k-Step Reach	$R_v^k(G) = \left\lvert\{u \in V \setminus \{v\} : dist_{u,v}(G) \leq k\}\right\rvert$
Eccentricity	$EC_v(G) = \max_{u \in V}\{dist_{u,v}(G)\}$

Distance-based Centralities

$$\sum_{u \in V \setminus \{v\}} a_{dist_{u,v}(G)}$$

Alternatives:

ADDITIVE (Harmonic, Decay, Degree, k-Step Reach)

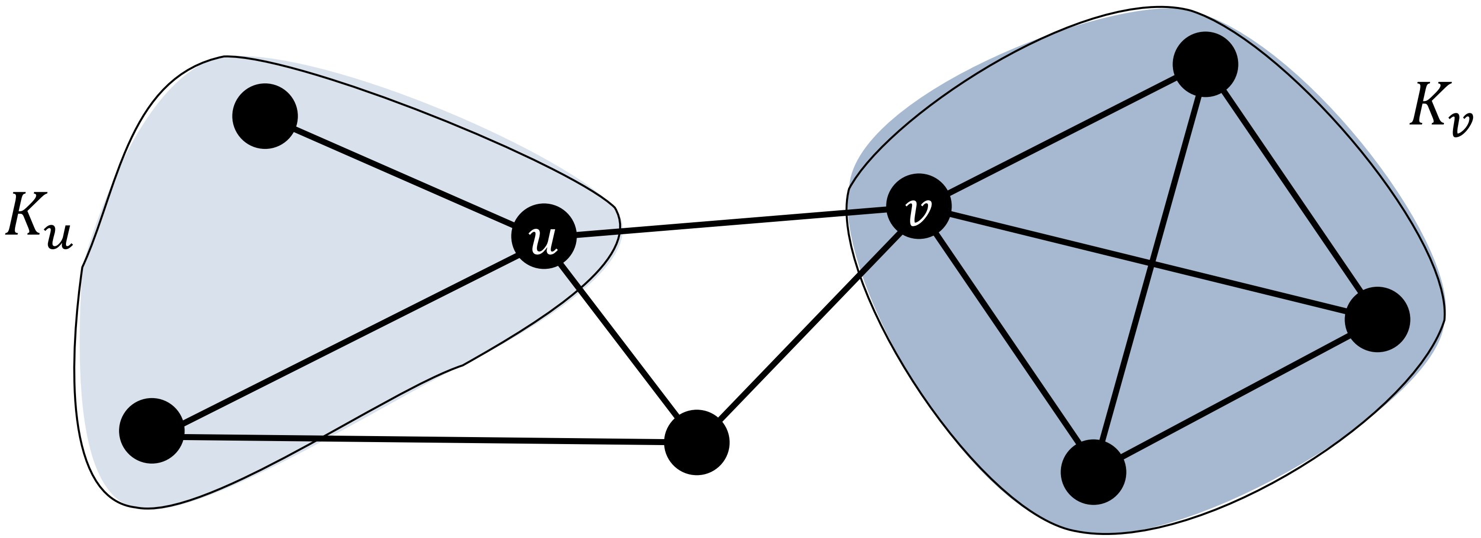
		a_1	a_2	a_3	$a_{i>3}$
Harmonic [Rochat 2009]	$H_v(G) = \sum_u 1/dist_{u,v}(G)$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{i}$
Decay [Jackson 2005]	$Y_v(G) = \sum_u \delta^{dist_{u,v}(G)}$ where $\delta \in (0,1)$	δ^0	δ^1	δ^2	δ^i
Degree	$D_v(G) = \lvert\{u \in V \setminus \{v\} : (u,v) \in V\}\rvert$	1	0	0	0
k-Step Reach	$R_v^k(G) = \left\lvert\{u \in V \setminus \{v\} : dist_{u,v}(G) \leq k\}\right\rvert$	1	1	1	$[i \leq k]$
Eccentricity	$EC_v(G) = \max_{u \in V}\{dist_{u,v}(G)\}$				

Closeness Centrality [Sabidussi 1966]

Finds the center of a graph.

$$C_v(G) = \frac{1}{\sum_u dist_{u,v}(G)}$$

Note: well defined only for (strongly) connected graphs



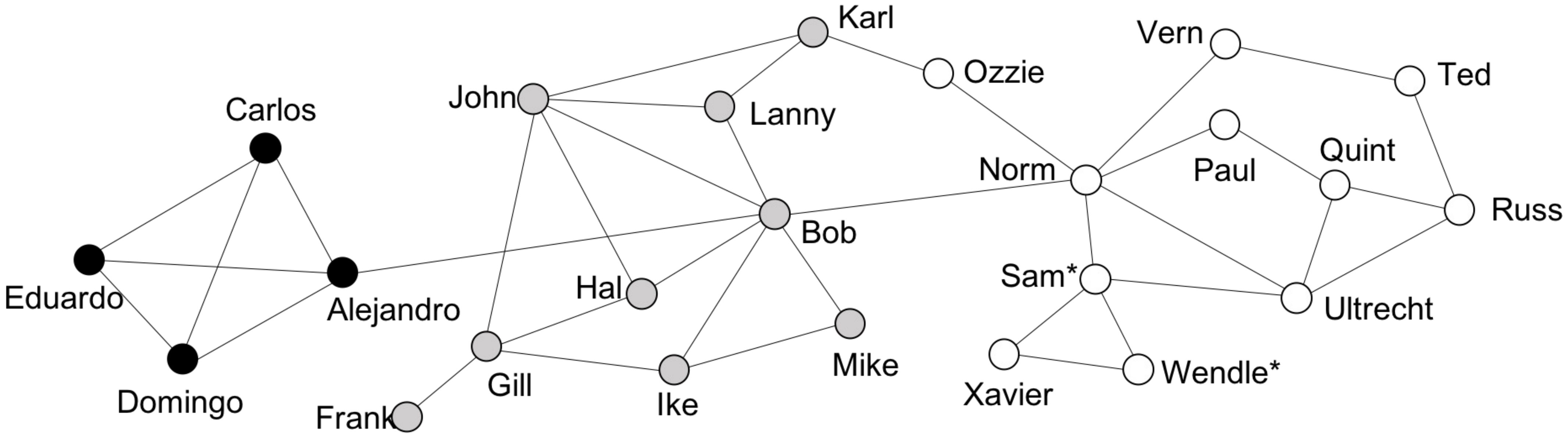
If K_u is the set of nodes closer to u than to v and K_v - closer to v than to u, then

$$(|K_u| \leq |K_v|) \Leftrightarrow (C_u(G) \leq C_v(G))$$

Group Centrality Measures

Group of 24 workers on strike with two negotiators: Sam and Wendle

Management hired an external consultant for help…



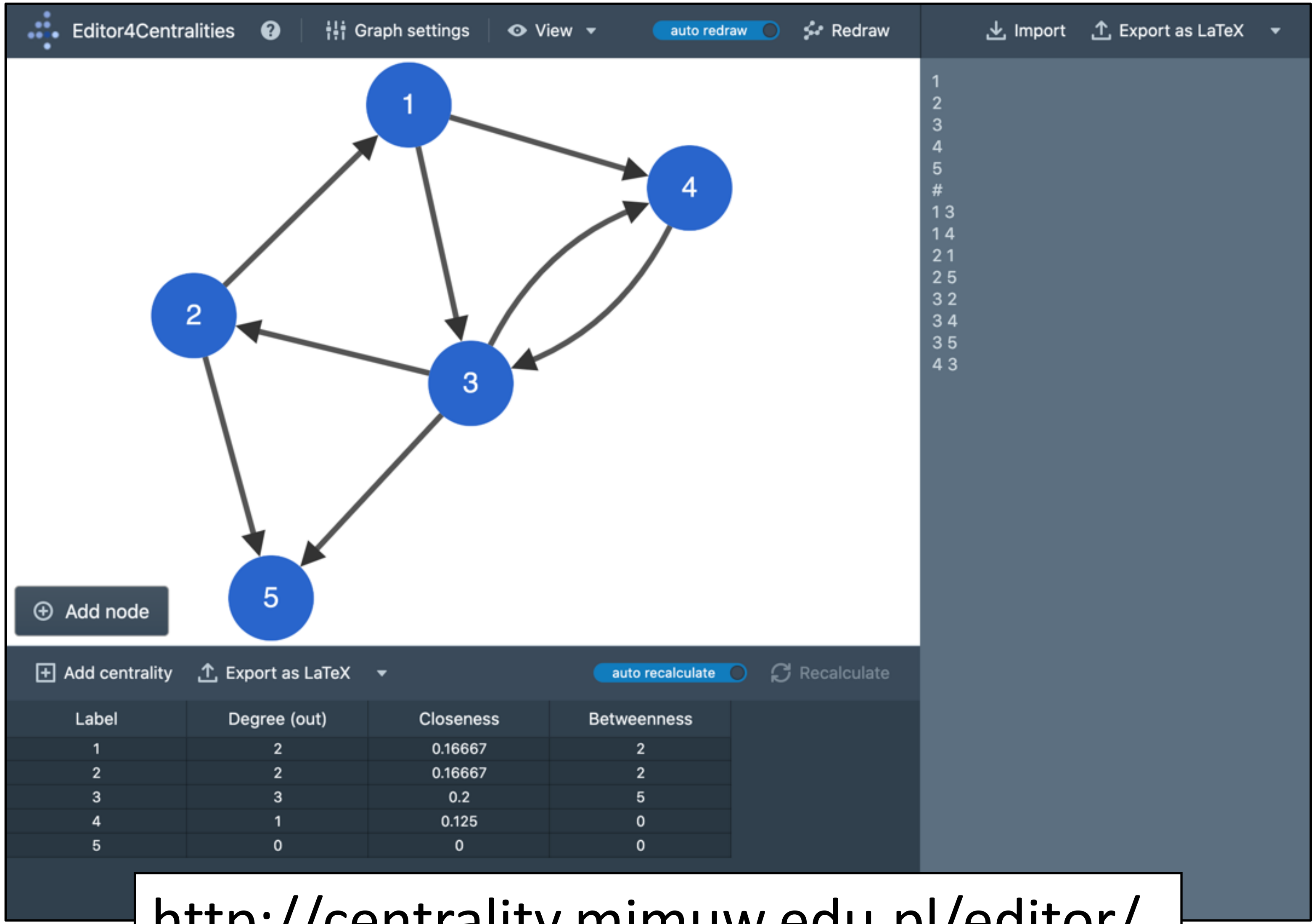
Judd H Michael. Labor dispute reconciliation in a forest products manufacturing facility. *Forest products journal*, 47(11/12):41, 1997.

!!! TEMAT !!!

Reguły wyborów komitetowych jako grupowe miary centralności

Implementacja + analiza (teoretyczna lub eksperymentalna)

Editor4Centralities - free online tool



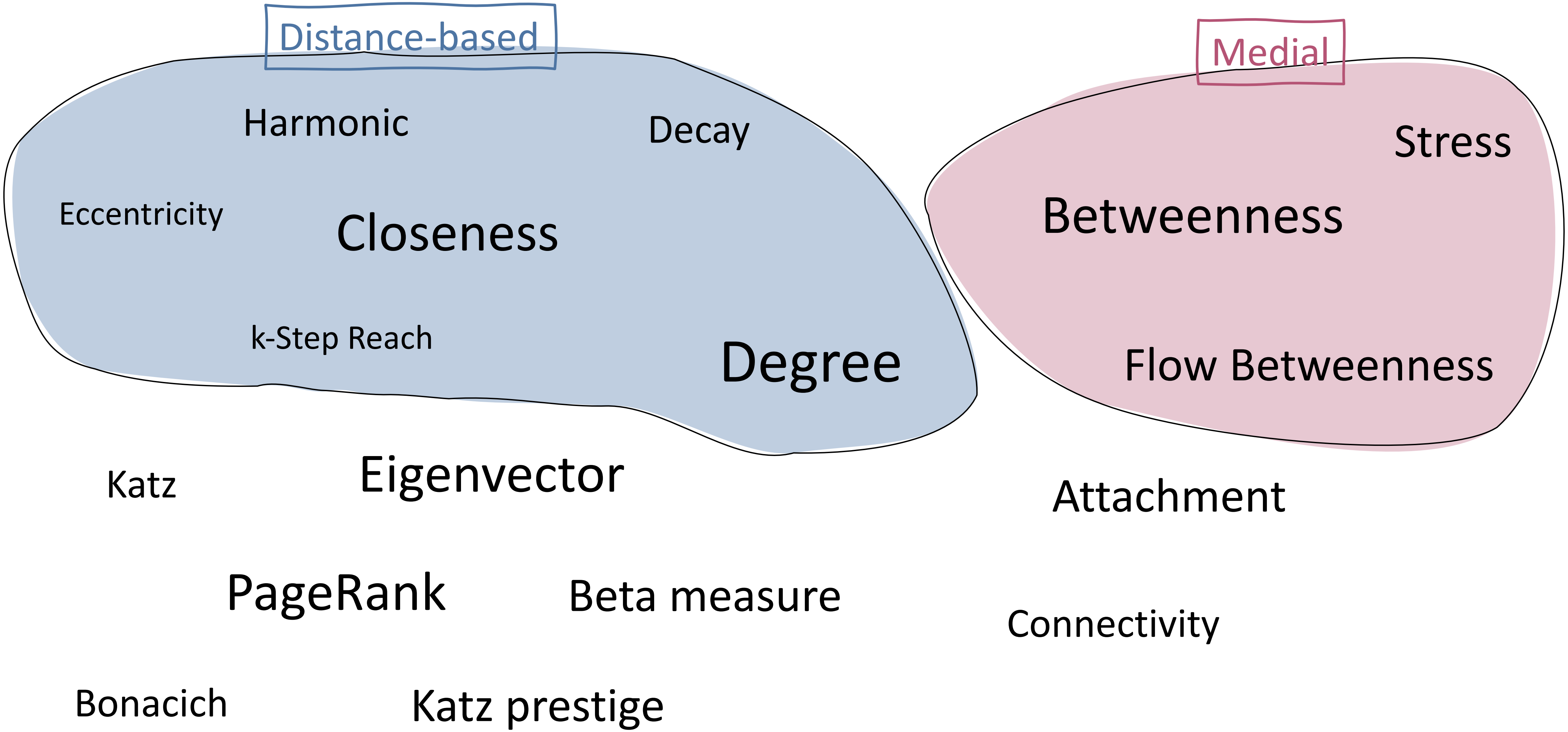
http://centrality.mimuw.edu.pl/editor/

Calculates 21 centralities

Graph can be created in text form

Export simple graphs to latex

Medial Centralities



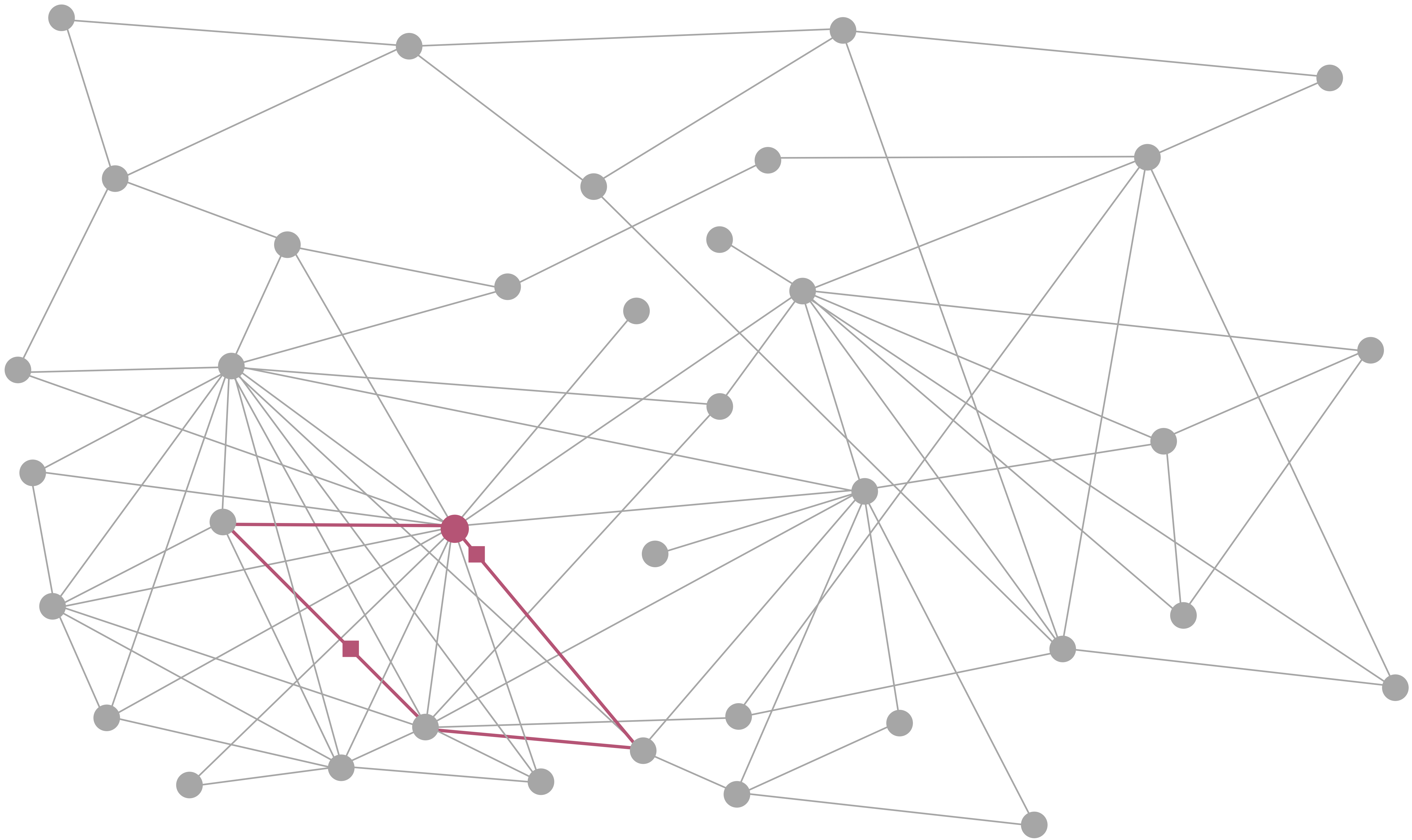
Medial Centralities

Assess a node by the role as an intermediary between other nodes.

LOOSE DEFINITION

A centrality measure is a *medial centrality* if there exists a function $\Delta_v^{s,t}(G)$ that evaluates the role of v in connecting s, t in G such that

$$F_v(G) = \sum_{s,t \in V} \Delta_v^{s,t}(G).$$



Betweenness Centrality [Freeman 1977]

Finds the node through which the most information passes.

$$B_v(G) = \sum_{s,t \in V \setminus \{v\}} \frac{\delta_{s,t}(v)}{\delta_{s,t}}$$

where $\delta_{s,t}$ is the number of the shortest paths from s to t and $\delta_{s,t}(v)$ is the number of these shortest paths that go through v.

- this is a „competitive" centrality measure – when we add an edge, some nodes gain, but some lose
- it finds gatekeepers, but also nodes with high degree

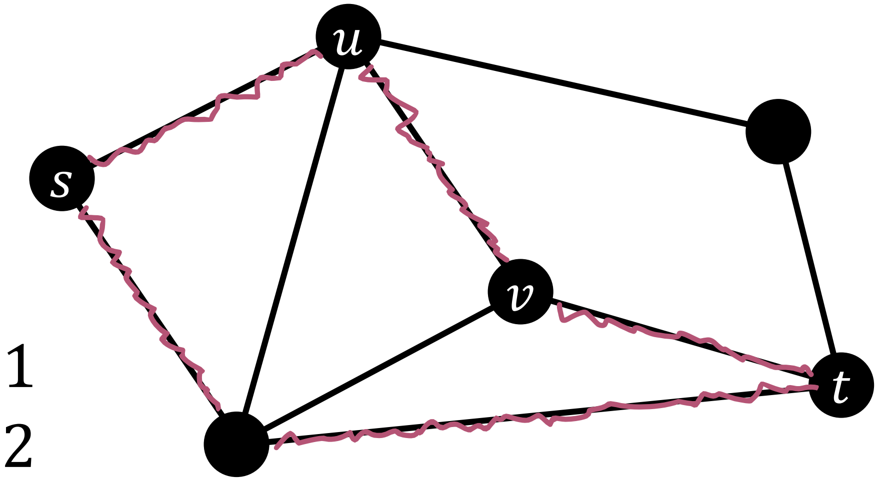
Medial Centralities

Alternatives:

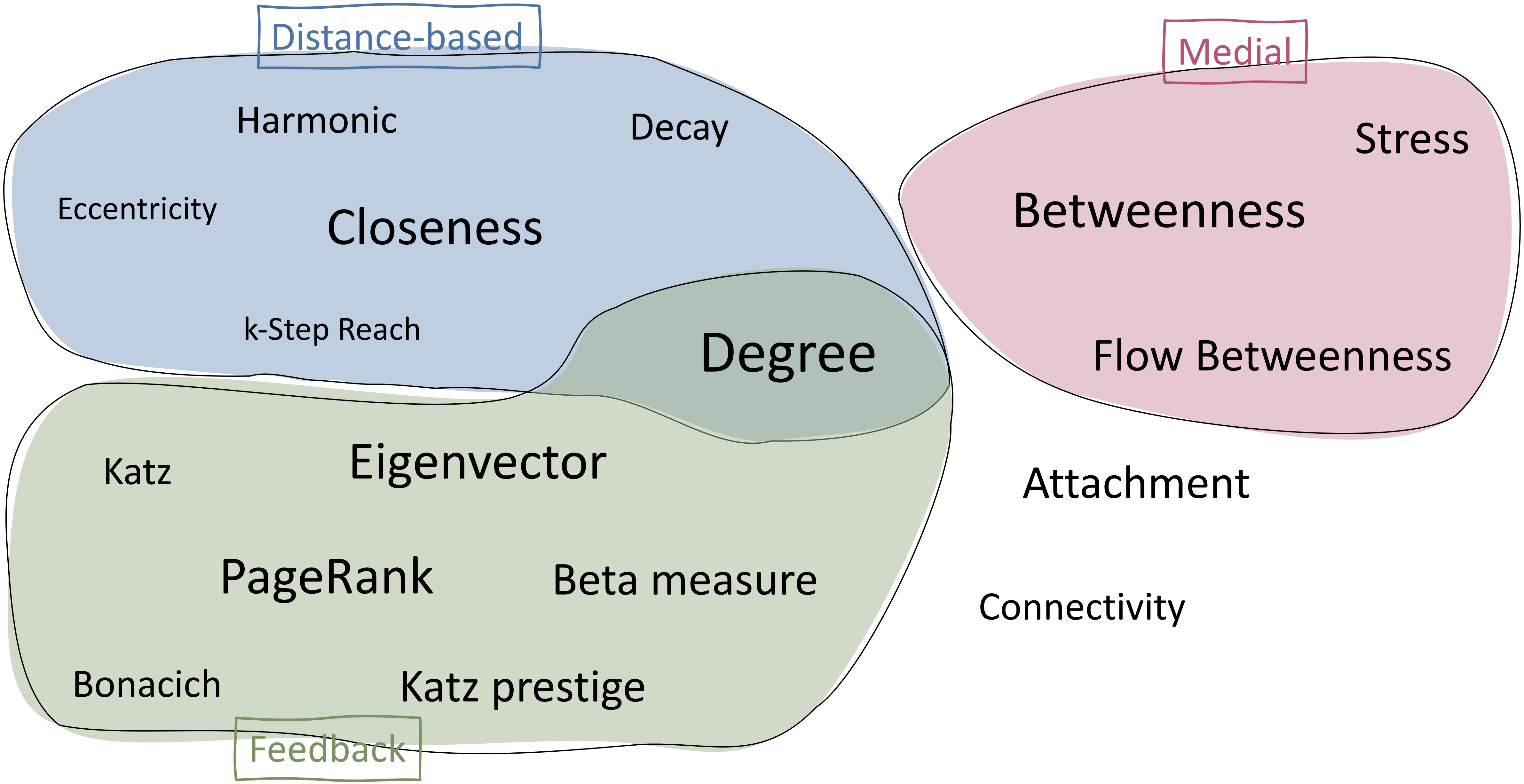
Stress Centrality [Shimbel 1953]	$S_v(G) = \sum_{s,t \in V} \delta_{s,t}(v)$
Flow Betweenness Centrality [Freeman et al. 1991]	$FB_v(G) = \sum_{s,t \in V} flow_{s,t}(G) - flow_{s,t}(G - v)$

$flow_{s,t}(G)$ for unweighted graphs is simply the number of edge-disjoint paths from s to t

$$flow_{s,t}(G) = 2$$
$$flow_{s,t}(G - u) = 1$$
$$flow_{s,t}(G - v) = 2$$



Feedback Centralities

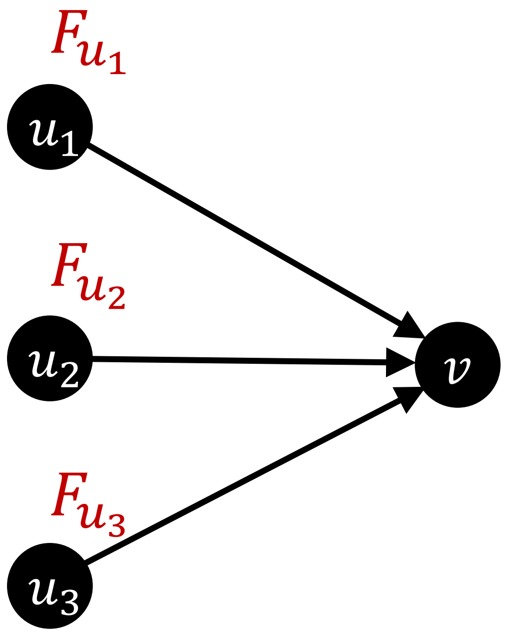


Feedback Centralities

Assess a node by the importance of its neighbors (direct predecessors).

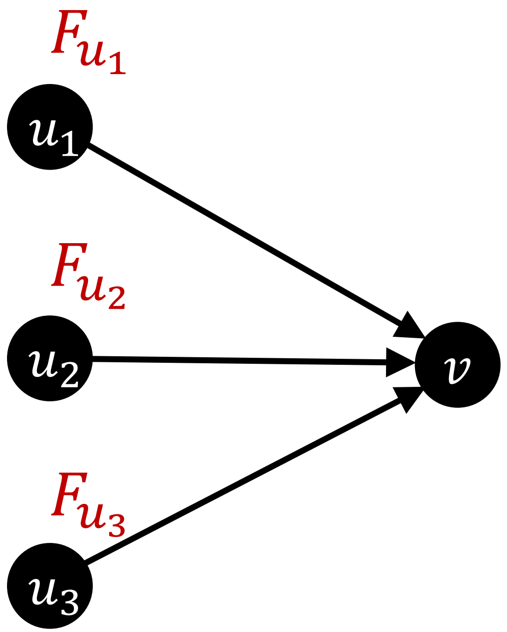
LOOSE DEFINITION

A centrality measure is a *feedback centrality* if the centrality of a node (mostly) depends on the centralities and out-degrees of its predecessors.



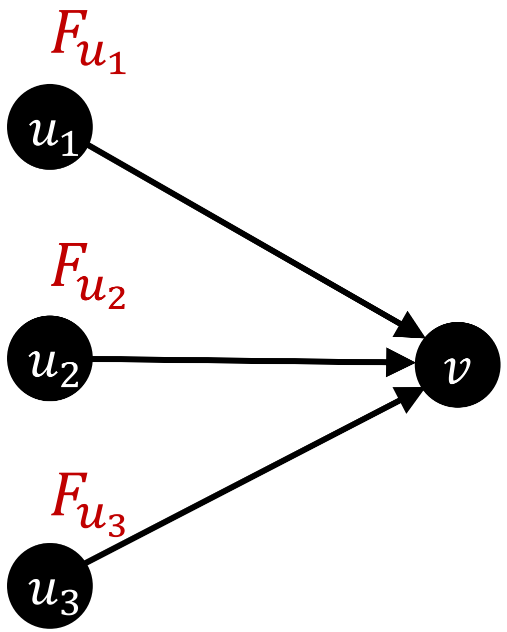
Feedback Centralities

Parallel	Distributed



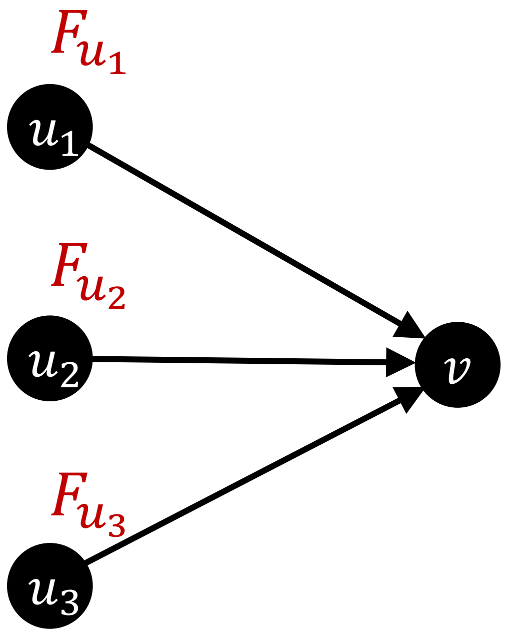
Feedback Centralities

Parallel	Distributed
(in-)Degree Centrality $F_v = 1 + 1 + 1$	



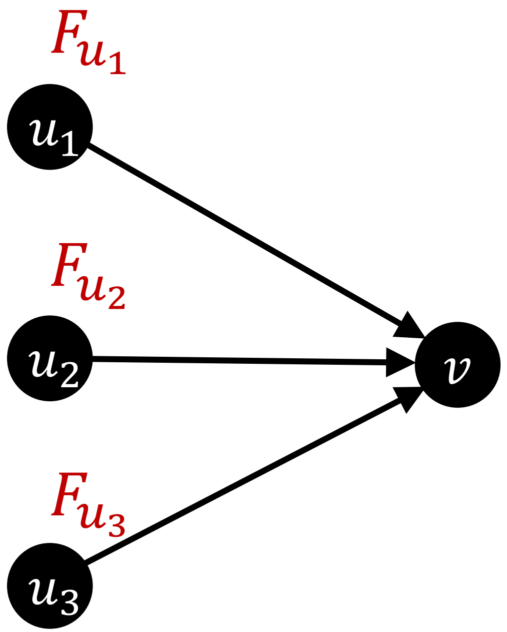
Feedback Centralities

Parallel	Distributed
(in-)Degree Centrality $F_v = 1 + 1 + 1$	
Eigenvector Centrality $F_v = \frac{1}{\lambda}(F_{u_1} + F_{u_2} + F_{u_3})$	



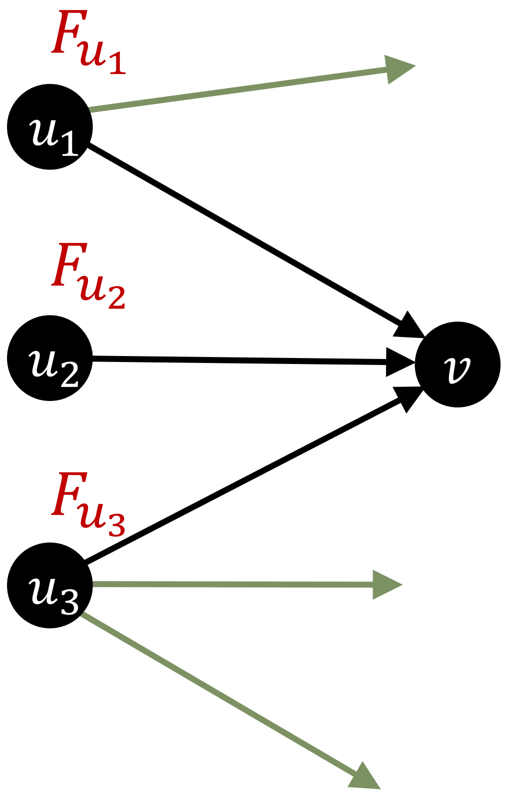
Feedback Centralities

Parallel	Distributed
(in-)Degree Centrality $F_v = 1 + 1 + 1$	
Eigenvector Centrality $F_v = \frac{1}{\lambda}\left(F_{u_1} + F_{u_2} + F_{u_3}\right)$	
Katz Centrality $F_v = a\left(F_{u_1} + F_{u_2} + F_{u_3}\right) + b$	



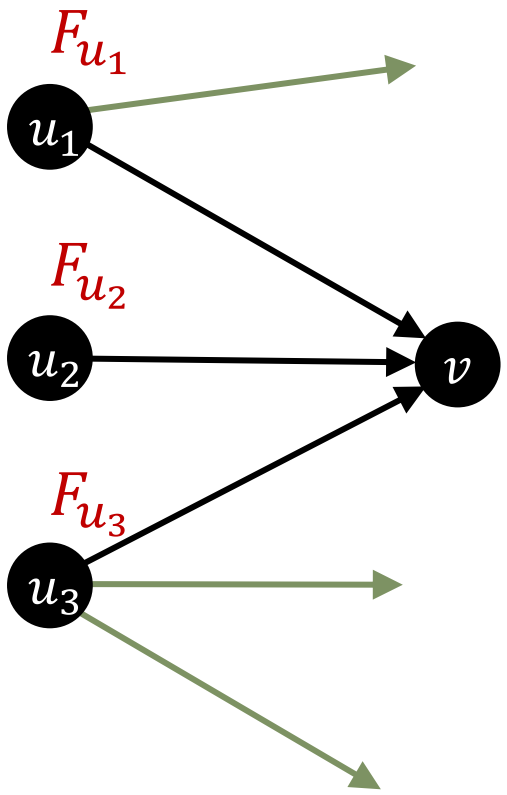
Feedback Centralities

Parallel	Distributed
(in-)Degree Centrality $F_v = 1 + 1 + 1$	
Eigenvector Centrality $F_v = \frac{1}{\lambda}(F_{u_1} + F_{u_2} + F_{u_3})$	
Katz Centrality $F_v = a\left(F_{u_1} + F_{u_2} + F_{u_3}\right) + b$	



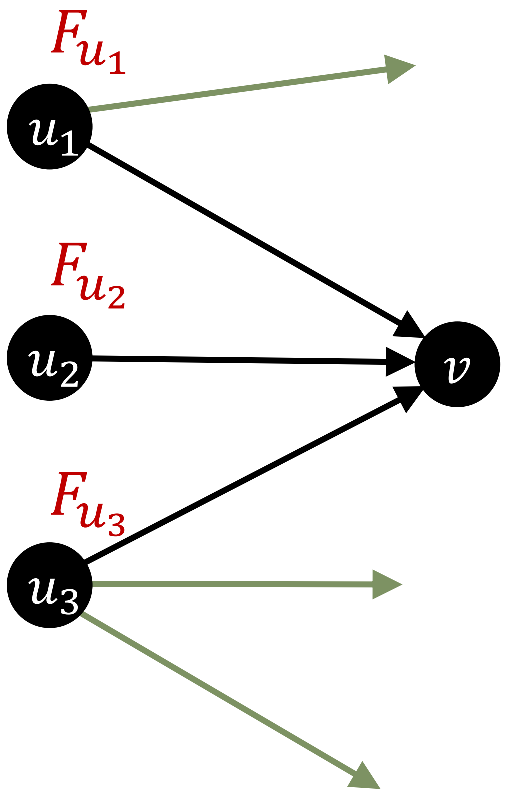
Feedback Centralities

Parallel	Distributed
(in-)Degree Centrality $F_v = 1 + 1 + 1$	Beta Measure $F_v = \frac{1}{2} + \frac{1}{1} + \frac{1}{3}$
Eigenvector Centrality $F_v = \frac{1}{\lambda}(F_{u_1} + F_{u_2} + F_{u_3})$	
Katz Centrality $F_v = a\left(F_{u_1} + F_{u_2} + F_{u_3}\right) + b$	



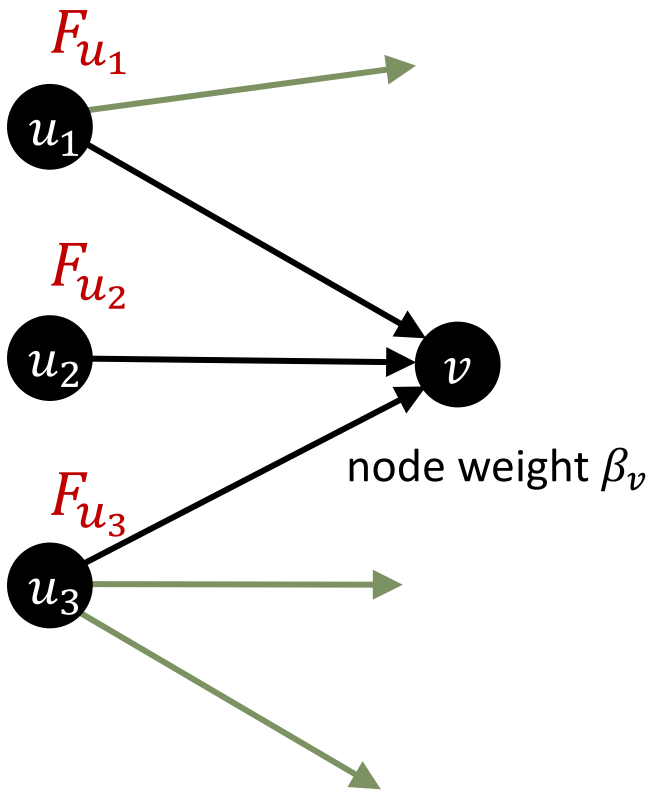
Feedback Centralities

Parallel	Distributed
(in-)Degree Centrality $F_v = 1 + 1 + 1$	Beta Measure $F_v = \frac{1}{2} + \frac{1}{1} + \frac{1}{3}$
Eigenvector Centrality $F_v = \frac{1}{\lambda}(F_{u_1} + F_{u_2} + F_{u_3})$	Katz Prestige $F_v = \frac{F_{u_1}}{2} + \frac{F_{u_2}}{1} + \frac{F_{u_3}}{3}$
Katz Centrality $F_v = a\left(F_{u_1} + F_{u_2} + F_{u_3}\right) + b$	



Feedback Centralities

Parallel	Distributed
(in-)Degree Centrality $F_v = 1 + 1 + 1$	Beta Measure $F_v = \frac{1}{2} + \frac{1}{1} + \frac{1}{3}$
Eigenvector Centrality $F_v = \frac{1}{\lambda}(F_{u_1} + F_{u_2} + F_{u_3})$	Katz Prestige $F_v = \frac{F_{u_1}}{2} + \frac{F_{u_2}}{1} + \frac{F_{u_3}}{3}$
Katz Centrality $F_v = a\,(F_{u_1} + F_{u_2} + F_{u_3}) + b$	PageRank $F_v = a\left(\frac{F_{u_1}}{2} + \frac{F_{u_2}}{1} + \frac{F_{u_3}}{3}\right) + \beta_v$



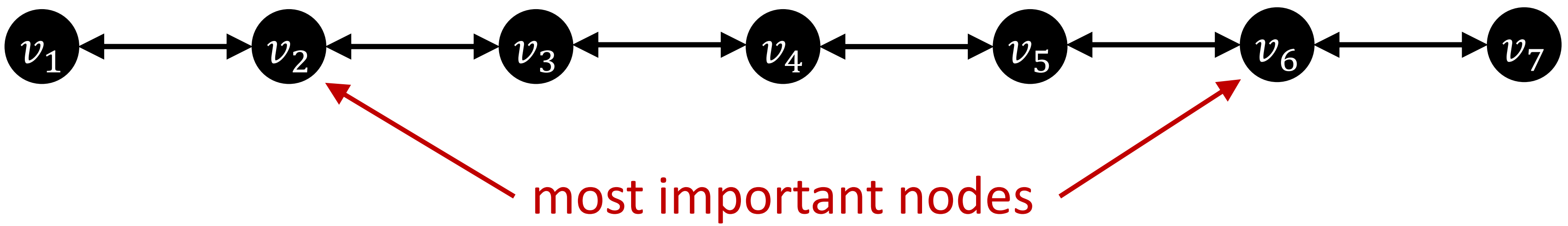
Feedback Centralities

λ is the largest eigenvalue of the adjacency matrix

Eigenvector Centrality [Bonacich 1972]	$E_v(G) = \frac{1}{\lambda}\sum_{(u,v)\in E} E_u(G)$	only for (strongly) connected graphs
Katz Centrality [Katz 1953]	$K_v(G) = a\left(\sum_{(u,v)\in E} K_u(G)\right) + b$	decay factor $a < 1/\lambda$
Bonacich Centrality [Bonacich 1987]	$BK_v(G) = \sum_{(u,v)\in E}(a \cdot BK_u(G) + b)$	$= \frac{1}{a}(K(G) - b \cdot \mathbf{1})$
Beta Measure [van den Brink 1994]	$\beta M_v(G) = \sum_{(u,v)\in E} \frac{1}{\deg_u^+(G)}$	$\deg_u^+(G)$ = out-degree of node u
Katz Prestige (or Seeley Index) [Seeley 1949]	$KP_v(G) = \sum_{(u,v)\in E} \frac{KP_u(G)}{\deg_u^+(G)}$	only for (strongly) connected graphs
PageRank [Page et al. 1999]	$PR_v(G) = a\left(\sum_{(u,v)\in E} \frac{PR_u(G)}{\deg_u^+(G)}\right) + \beta_v$	usually $a \geq 0.8$ β_v is weight of v

Reasons to use PageRank

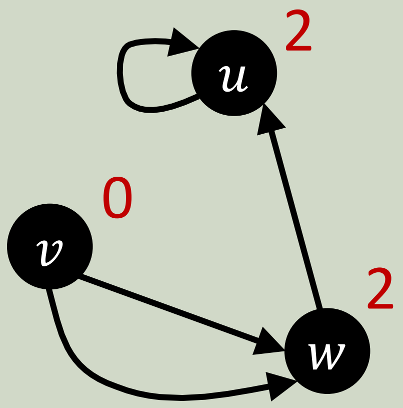
- it can be computed fast
- it is popular
- it works for the World Wide Web
- it makes sense intuitively
- it gives results that we wanted to achieve
- it is complicated so it has to do something smart



Model of the Web

We consider *directed multigraphs* with *node weights*.

- set of nodes V
- multiset of edges $E \subseteq V \times V$
- node weights $\beta: V \to \mathbb{R}_{\geq 0}$



EXAMPLE

$V = \{u, v, w\}$
$E = \{(v, w), (v, w), (w, u), (u, u)\}$
$\beta = (\beta_u, \beta_v, \beta_w) = (0,1,0)$

Centrality measure is a function that given a graph $G = (V, E, \beta)$ and a node $v \in V$ returns a real value $F_v(G) \in \mathbb{R}_{\geq 0}$.

E.g., (in-)Degree is defined as follows: $F_v(G) = |\{(u, v) \in E : u \in V\}|$

Axioms for PageRank

Node Deletion

Edge Multiplication

Node Redirect

Edge Deletion

Edge Swap

Baseline

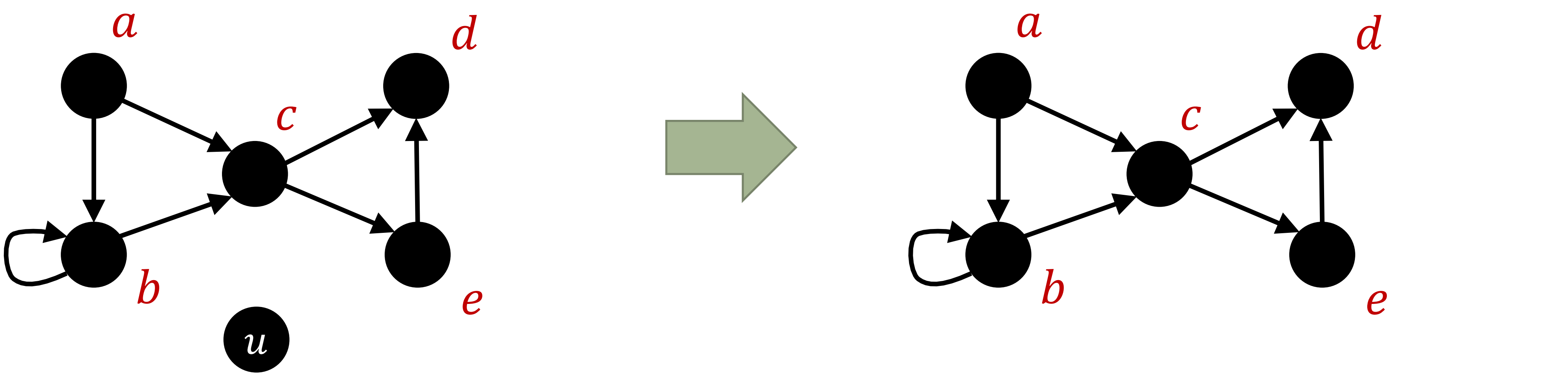
Invariance axioms

Axiom 1: Node Deletion

For every graph $G = (V, E, \beta)$ and isolated node $u \in V$:

$$F_v(G) = F_v(G - u)$$

for every node $v \in V \setminus \{u\}$.



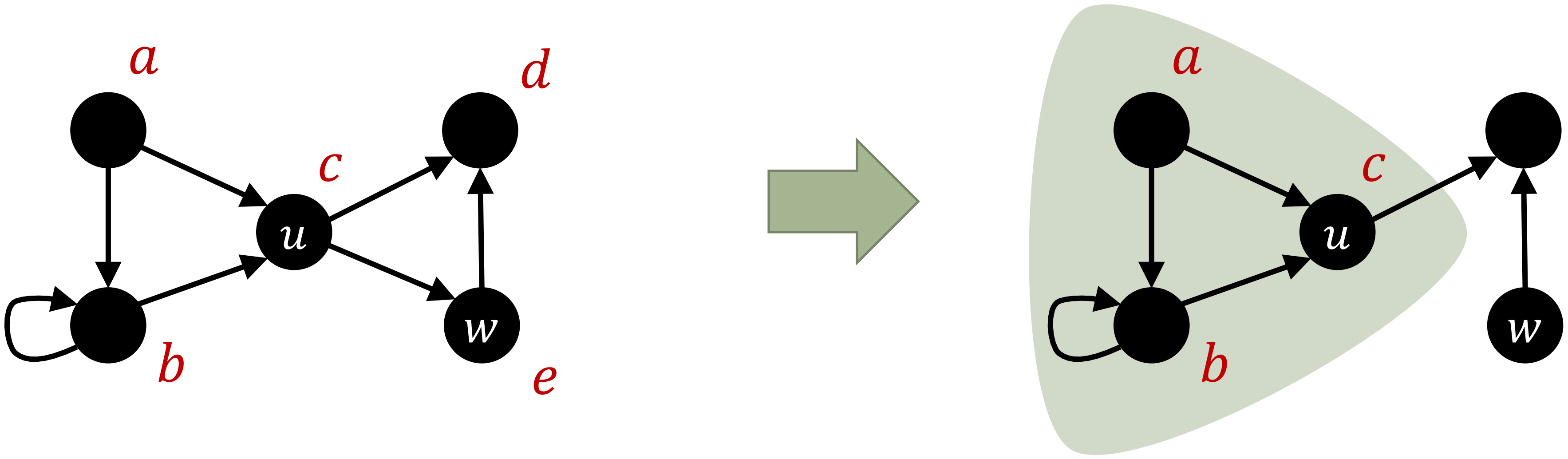
TL;DR: Removing an isolated node does not affect the centrality of any other node.

Axiom 2: Edge Deletion

For every graph $G = (V, E, \beta)$ and edge $(u, w) \in E$:

$$F_v(G) = F_v(G - (u, w))$$

for every node $v \in V$ which is not a successor of u.



TL;DR: Removing an edge from the graph does not affect the centrality of any node which is not a successor of the start of this edge.

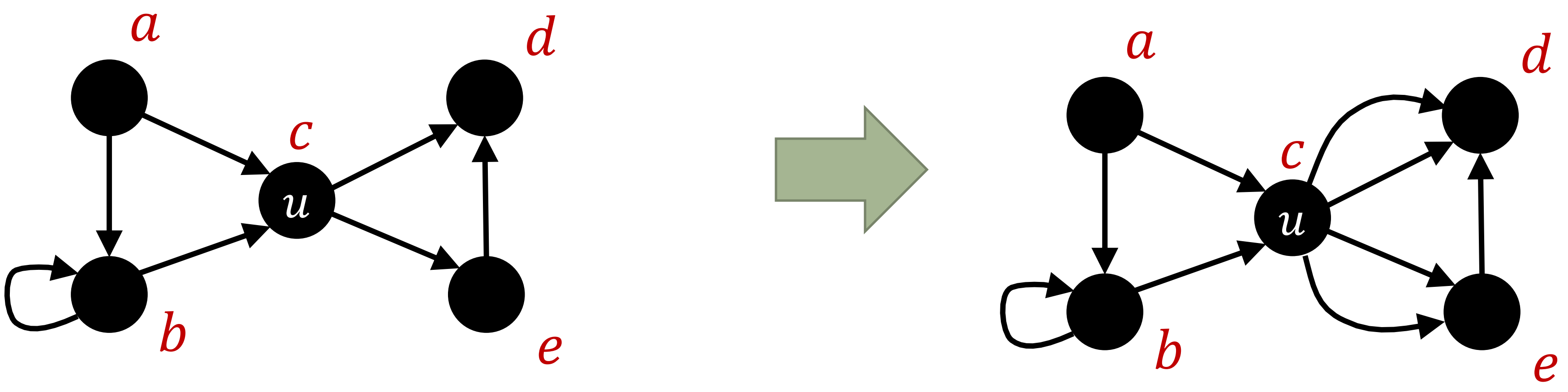
Axiom 3: Edge Multiplication

For every graph $G = (V, E, \beta)$, node $u \in V$ and $k \in \mathbb{N}$:

$$F_v(G) = F_v\big(G + \underbrace{\Gamma_u^+(G) + \cdots + \Gamma_u^+(G)}_{k}\big)$$

outgoing edges of u

for every node $v \in V$.



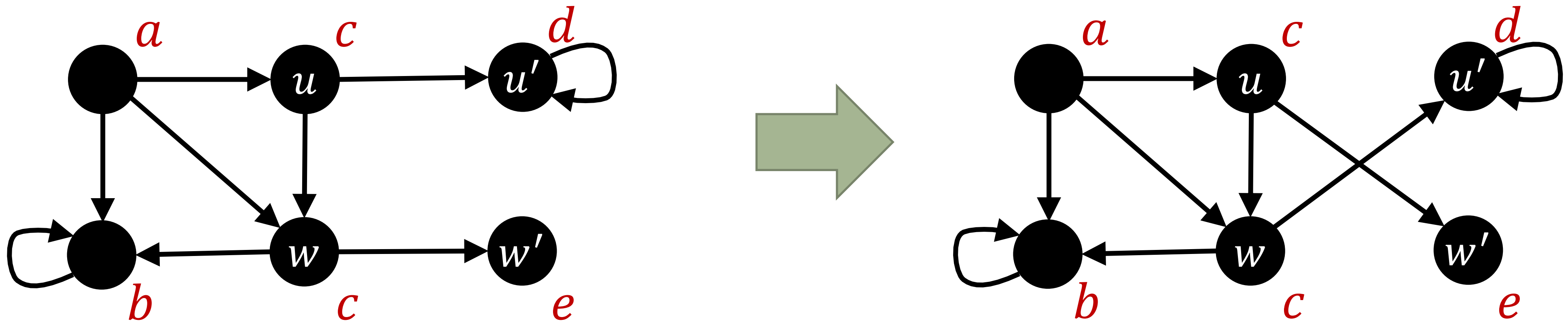
TL;DR: Creating additional copies of the outgoing edges of a node does not affect the centrality of any node.

Axiom 4: Edge Swap

For every graph $G = (V, E, \beta)$ and edges $(u, u'), (w, w') \in E$ s.t. $F_u(G) = F_w(G)$ and $|\Gamma_u^+(G)| = |\Gamma_w^+(G)|$:

$$F_v(G) = F_v\big(G - (u, u') - (w, w') + (u, w') + (w, u')\big)$$

for every node $v \in V$.



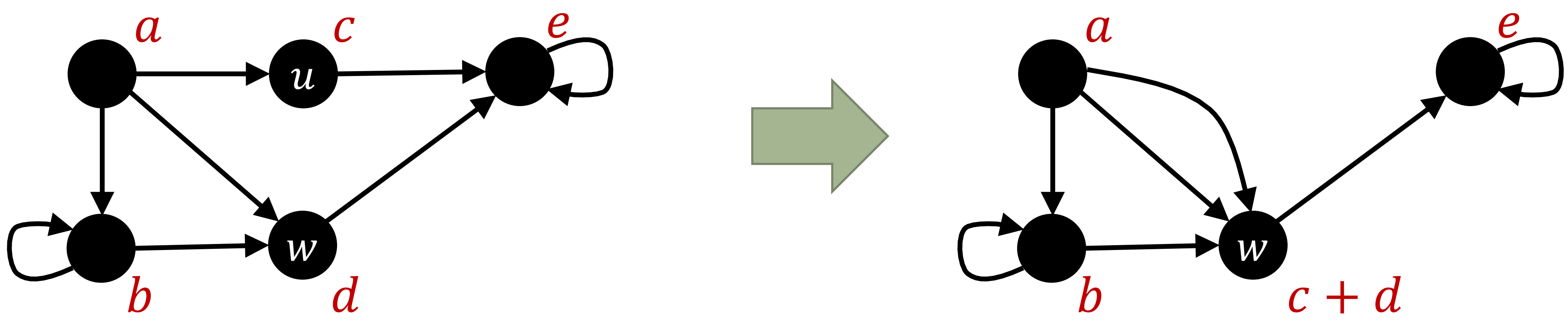
TL;DR: Swapping ends of two outgoing edges of nodes with equal centralities and out-degrees does not affect the centrality of any node.

Axiom 5: Node Redirect

For every graph $G = (V, E, \beta)$ and out-twins $u, w \in V$:

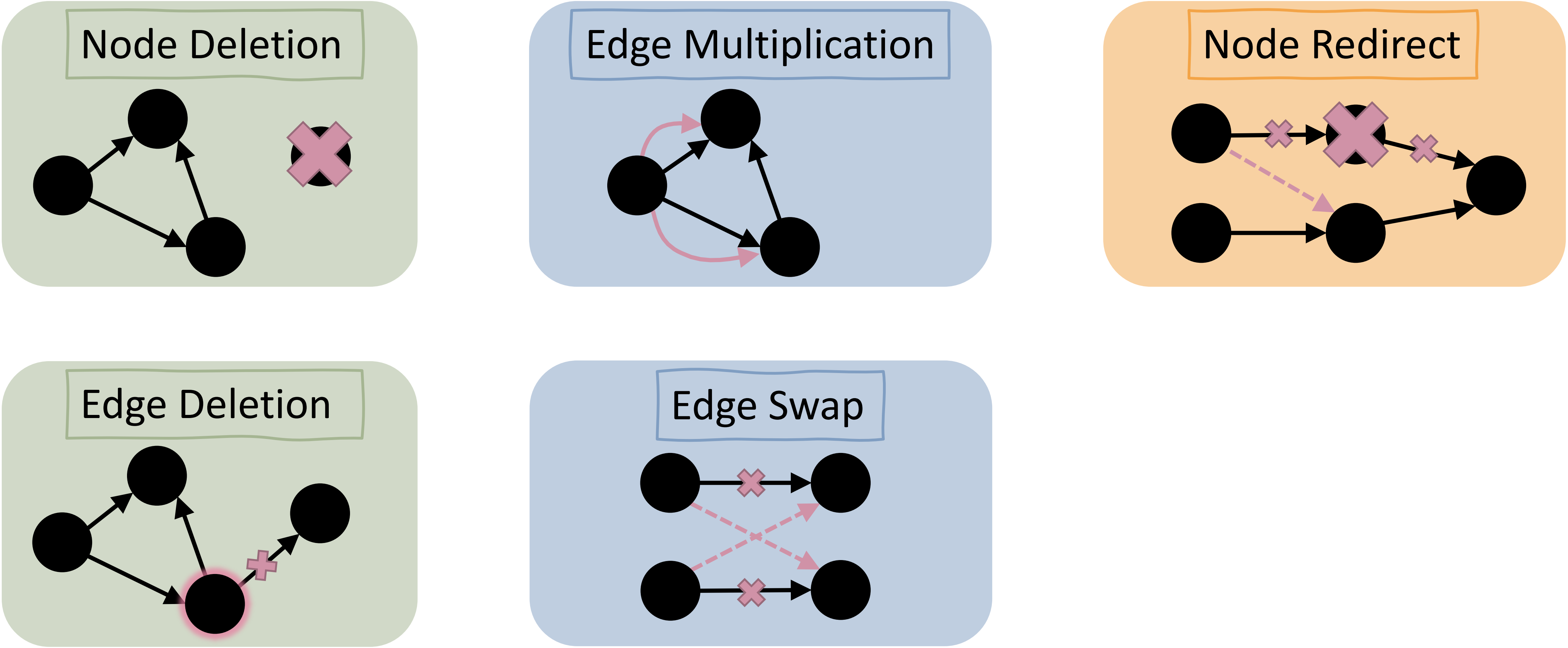
$$F_v\left(G \text{ with } u \xrightarrow{\text{REDIRECT}} w\right) = \begin{cases} F_v(G) & \text{if } v \in V \setminus \{u, w\}, \\ F_v(G) + F_u(G) & \text{otherwise,} \end{cases}$$

for every $v \in V \setminus \{u\}$.



TL;DR: Redirecting a node into out-twin sums up their centralities and does not affect the centrality of any other node.

If a centrality measure satisfies:

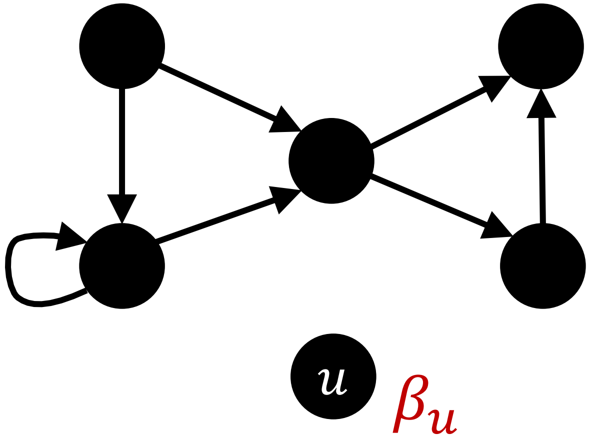


then it is PageRank (for some α) × some constant.

Axiom 6: Baseline

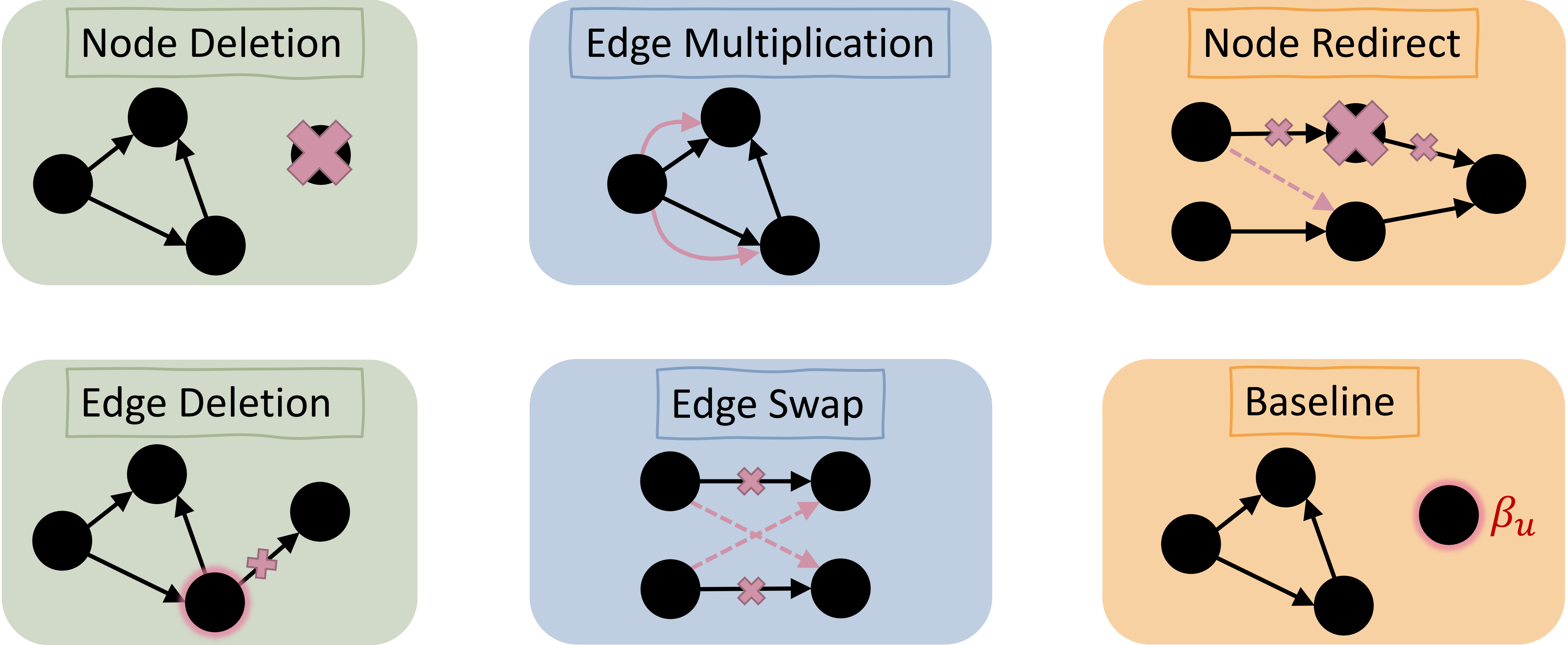
For every graph $G = (V, E, \beta)$ and isolated node $u \in V$:

$$F_u(G) = \beta_u.$$



TL;DR: The centrality of an isolated node is equal to its weight.

If a centrality measure satisfies:



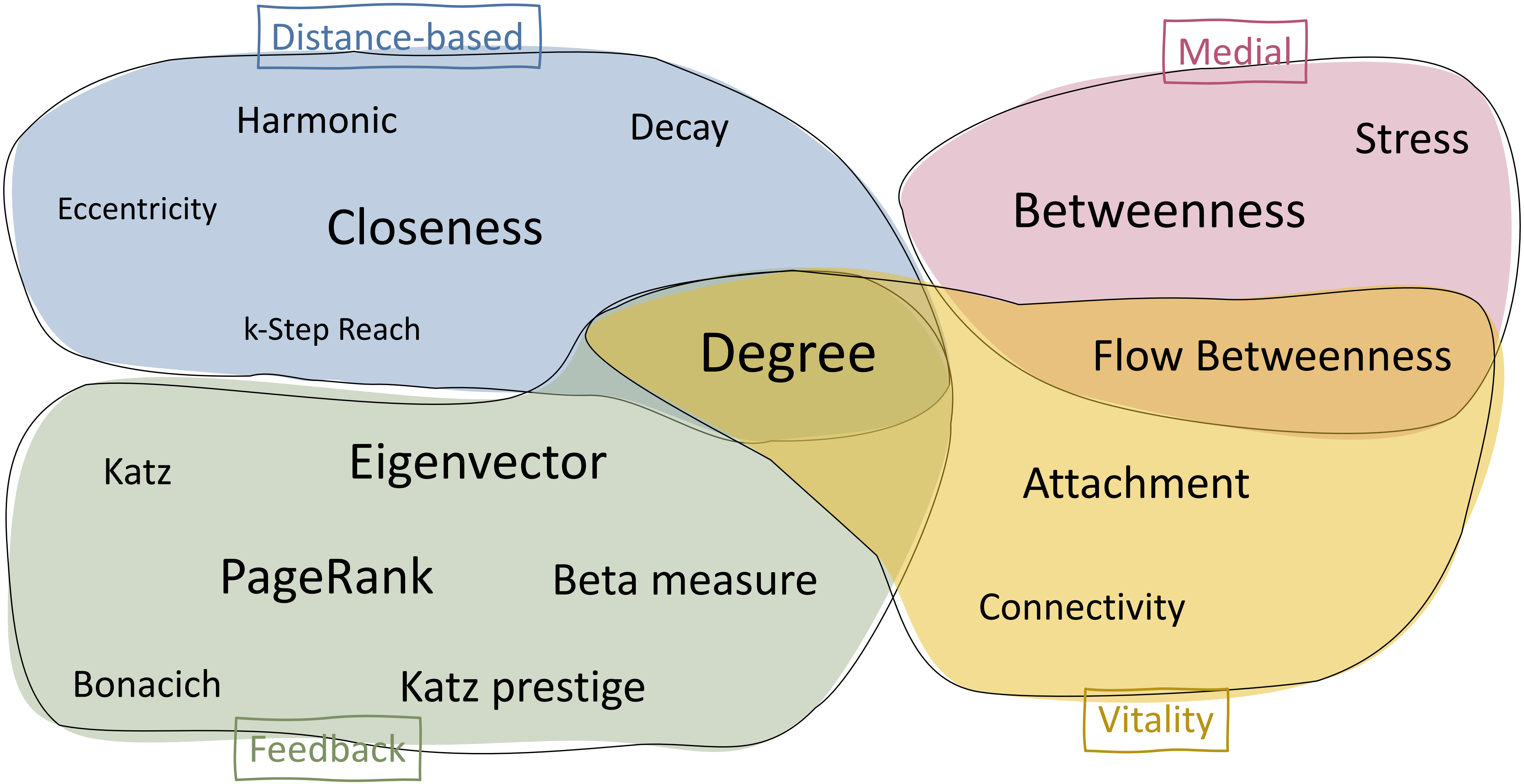
then it is PageRank (for some α).

<u>!!! TEMAT !!!</u>

Analiza miar centralności pod kątem sprawiedliwości

Analiza teoretyczna

Vitality Indices



Vitality Indices

Assess a node by the decrease in the quality of the network if the node was removed.

DEFINITION

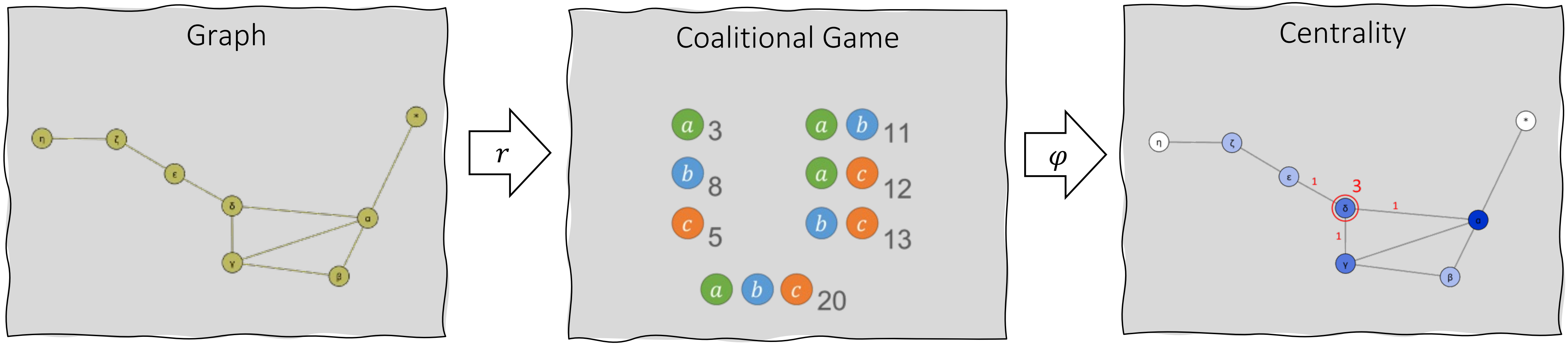
A centrality measure is a *vitality index* if there exists a function $f : \mathcal{G} \to \mathbb{R}$ such that

$$F_v(G) = f(G) - f(G - v)$$

Examples:

- for $f(G) = |E(G)|$ we get that Degree Centrality
- for $f(G) = \sum_{s,t \in V} flow_{s,t}(G)$ we get Flow Betweenness Centrality

Game-Theoretic Centralities



DEFINITION

Game-theoretic network centrality: (r, φ)

r: $G^V \to (2^V \to \mathbb{R})$ is a *representation function,*

φ: $(2^V \to \mathbb{R}) \to \mathbb{R}^V$ is a *solution concept.*

Game-Theoretic Centralities

φ is practically always the Shapley value.

Notation: $G[S] = (S, E[S])$ is a subgraph induced by S

What is r?

- Degree: $[r(G)](S) = 2 \cdot E[S]$
- Connectivity: $[r(G)](S) = 1$ if $G[S]$ is connected, $[r(G)](S) = 0$ oth. [Amer & Gimenez 2007]
- Attachment: $[r(G)](S) = 2(|S| - \text{number of components of } G[S])$ [Skibski et al. 2019]

In most existing definitions r is *induced:*

$$[r(G)](S) \text{ depends only on } G[S]$$

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

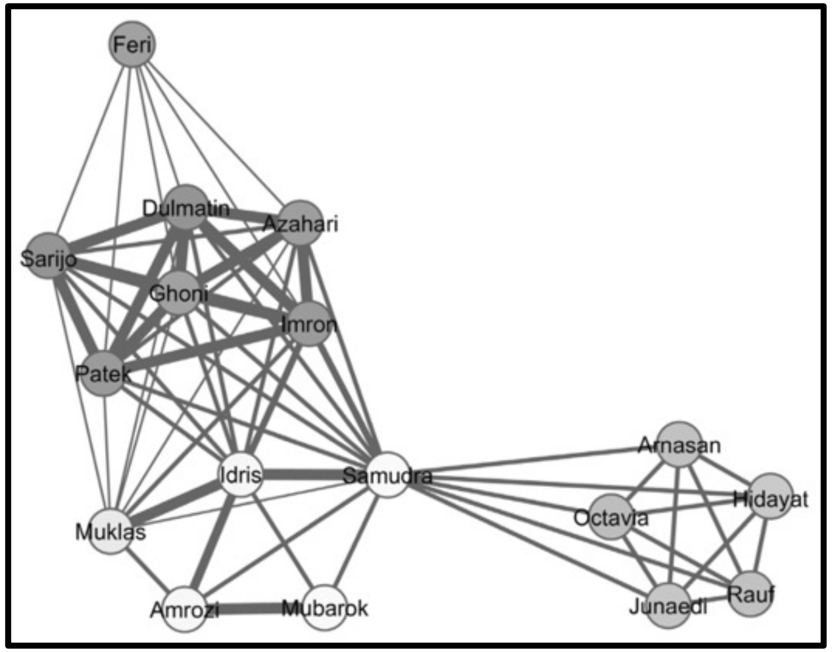
journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Cooperative game theoretic centrality analysis of terrorist networks: The cases of Jemaah Islamiyah and Al Qaeda

R.H.A. Lindelauf [a,b], H.J.M. Hamers [b], B.G.M. Husslage [b,c,*]

[a] Military Operational Art & Science, Netherlands Defense Academy, P.O. Box 90002, 4800 PA Breda, The Netherlands
[b] CentER and Department of Econometrics and Operations Research, Tilburg University, P.O. Box 90153, 5000 LE Tilburg, The Netherlands
[c] Department of Mathematics, Fontys University of Applied Sciences, P.O. Box 90900, 5000 GA Tilburg, The Netherlands



- Nodes = terrorists
- Edges = social ties
- representation function:

$$(r(G))(S) = \begin{cases} \frac{\sum_e 1}{\sum_e \omega(e)}, & \text{if } G[S] \text{ is connected,} \\ 0, & \text{otherwise.} \end{cases}$$

BIOINFORMATICS ORIGINAL PAPER
Vol. 26 no. 21 2010, pages 2721–2730
doi:10.1093/bioinformatics/btq508

Gene expression
Advance Access publication September 3, 2010

Using coalitional games on biological networks to measure centrality and power of genes

Stefano Moretti[1,2,*], Vito Fragnelli[3], Fioravante Patrone[4] and Stefano Bonassi[5]

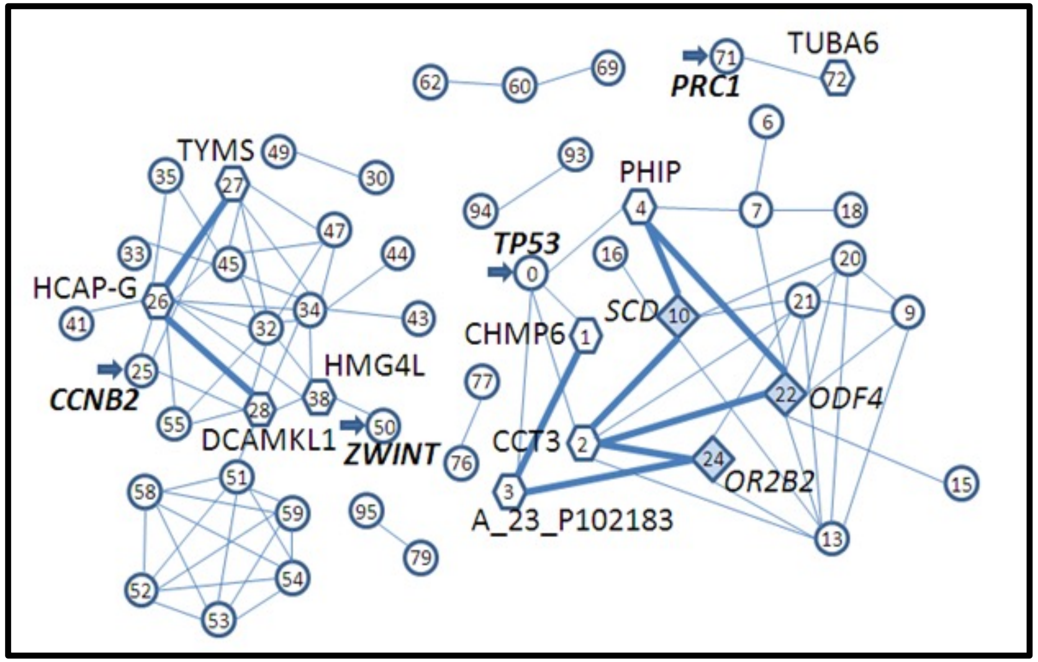
[1]CNRS, FRE3234, [2]Université Paris-Dauphine, Lamsade, F-75016 Paris, France, [3]Department of Advanced Sciences and Technologies, University of Eastern Piedmont, Alessandria, [4]DIPTEM, University of Genoa, Genoa and [5]Unit of Clinical and Molecular Epidemiology, IRCCS San Raffaele Pisana, Rome, Italy

Associate Editor: Trey Ideker

- Nodes = genes
- Edges = co-expression of genes
- 4 key genes associated with chromosome damage: ➡
- representation function:

$$f(S) = |\{key\ gene\ v : N(v) \subseteq S\}|$$

$$r_G(S) = \left(\sum_{C \in K(G[S])} f(C)\right) - f(S)$$

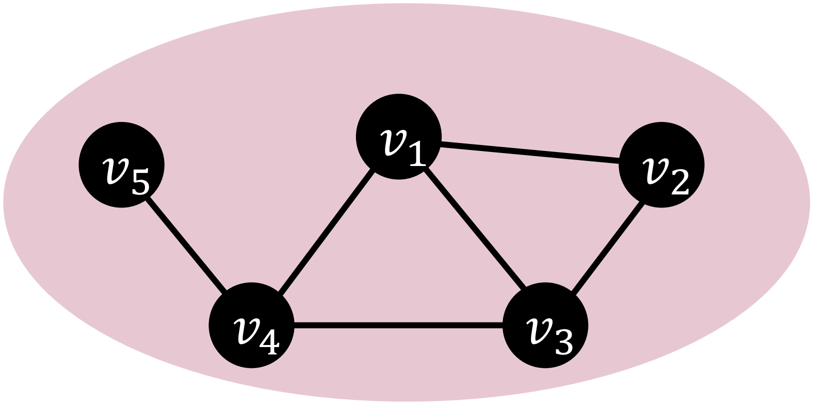


Vitality Indices = induced GTC

THEOREM

A centrality measure is a vitality index if and only if it is a Shapley-value based induced game-theoretic centralities.

How to extend node centralities to groups?



What is the degree centrality of $S = \{v_1, v_4\}$?

- $D_S(G) = 3 + 3 = 6$ (sum centralities)
- $D_S(G) = 3$ (merge the group)
- $D_S(G) = 2 + 2 = 4$ (count edges to others)

Group Vitality Indices

Assess a node by the decrease in the quality of the network if the <u>*group*</u> *is removed.*

DEFINITION

A group centrality measure is a *vitality index* if there exists a function $f: \mathcal{G} \to \mathbb{R}$ such that

$$F_S(G) = f(G) - f(G - S)$$

Examples:

- $D_S(G) = 5$ (we count edges incident to at least one node)!

Group Vitality Indices

FACT

For every node vitality index F', there exists a unique group vitality index F that extends it, i.e. $F_{\{v\}}(G) = F'_v(G)$ for every $G, v \in V(G)$.

Why?

For every $S \subseteq V, v \in S$:

$$F_S(G) = F_{\{v\}}(G) + F_{S \setminus \{v\}}(G - v)$$

Group Shapley Value

DEFINITION

The *group Shapley value* is defined as follows:

$$SV_S(N,v) = \sum_{T \subseteq N} \frac{(|T|-1)!\,(|N|-|T|)}{|N|!}\big(v(T) - v(T \setminus S)\big)$$

CZARNA SKRZYNKA

CZARNA
SKRZYNKA

Kolor	Uszy	Wąsy
żółty	okrągłe	TAK

CZARNA
SKRZYNKA

Kolor	Uszy	Wąsy
żółty	okrągłe	TAK

CZARNA SKRZYNKA

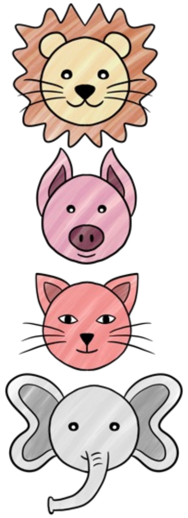
Wynik
100

	Kolor	Uszy	Wąsy		Wynik
	żółty	okrągłe	TAK		100
	różowy	spiczaste	NIE		30

CZARNA
SKRZYNKA

	Kolor	Uszy	Wąsy	Wynik
	żółty	okrągłe	TAK	100
	różowy	spiczaste	NIE	30
	czerwony	spiczaste	TAK	40

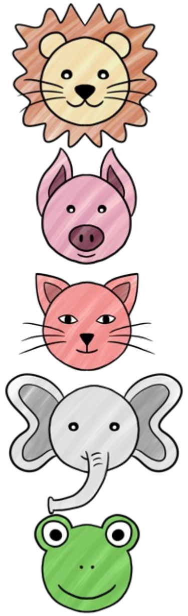
CZARNA
SKRZYNKA



Kolor	Uszy	Wąsy
żółty	okrągłe	TAK
różowy	spiczaste	NIE
czerwony	spiczaste	TAK
szary	falowane	NIE

CZARNA SKRZYNKA

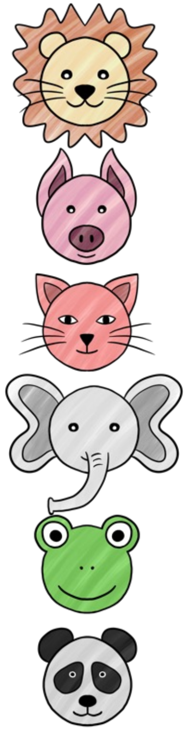
Wynik
100
30
40
70



Kolor	Uszy	Wąsy
żółty	okrągłe	TAK
różowy	spiczaste	NIE
czerwony	spiczaste	TAK
szary	falowane	NIE
zielony	–	NIE

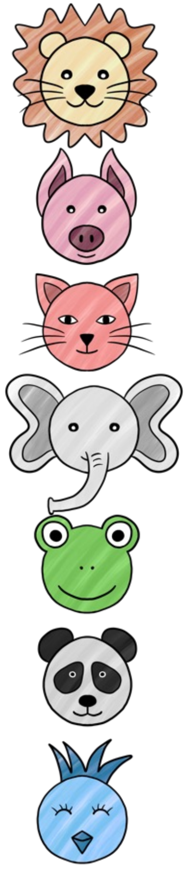
CZARNA SKRZYNKA

Wynik
100
30
40
70
10



Kolor	Uszy	Wąsy	Wynik
żółty	okrągłe	TAK	100
różowy	spiczaste	NIE	30
czerwony	spiczaste	TAK	40
szary	falowane	NIE	70
zielony	–	NIE	10
biało-czarny	okrągłe	NIE	0

CZARNA SKRZYNKA



Kolor	Uszy	Wąsy		Wynik
żółty	okrągłe	TAK	CZARNA SKRZYNKA	100
różowy	spiczaste	NIE		30
czerwony	spiczaste	TAK		40
szary	falowane	NIE		70
zielony	–	NIE		10
biało-czarny	okrągłe	NIE		0
niebieski	–	NIE		5

Kolor	Uszy	Wąsy
żółty	okrągłe	TAK

CZARNA
SKRZYNKA

Wynik
100

SHAP

Kolor	Uszy	Wąsy
żółty	okrągłe	TAK
żółty	okrągłe	–

CZARNA SKRZYNKA

Wynik
100
30

SHAP

Kolor	Uszy	Wąsy
żółty	okrągłe	TAK
żółty	okrągłe	–
żółty	–	TAK

CZARNA SKRZYNKA

Wynik
100
30
120

SHAP

Kolor	Uszy	Wąsy	Wynik
żółty	okrągłe	TAK	100
żółty	okrągłe	–	30
żółty	–	TAK	120
–	okrągłe	TAK	70

CZARNA SKRZYNKA

SHAP

Kolor	Uszy	Wąsy	Wynik
żółty	okrągłe	TAK	100
żółty	okrągłe	–	30
żółty	–	TAK	120
–	okrągłe	TAK	70
żółty	–	–	40

CZARNA SKRZYNKA

SHAP

Kolor	Uszy	Wąsy	Wynik
żółty	okrągłe	TAK	100
żółty	okrągłe	–	30
żółty	–	TAK	120
–	okrągłe	TAK	70
żółty	–	–	40
–	okrągłe	–	10

CZARNA SKRZYNKA

SHAP

Kolor	Uszy	Wąsy	Wynik
żółty	okrągłe	TAK	100
żółty	okrągłe	–	30
żółty	–	TAK	120
–	okrągłe	TAK	70
żółty	–	–	40
–	okrągłe	–	10
–	–	TAK	40

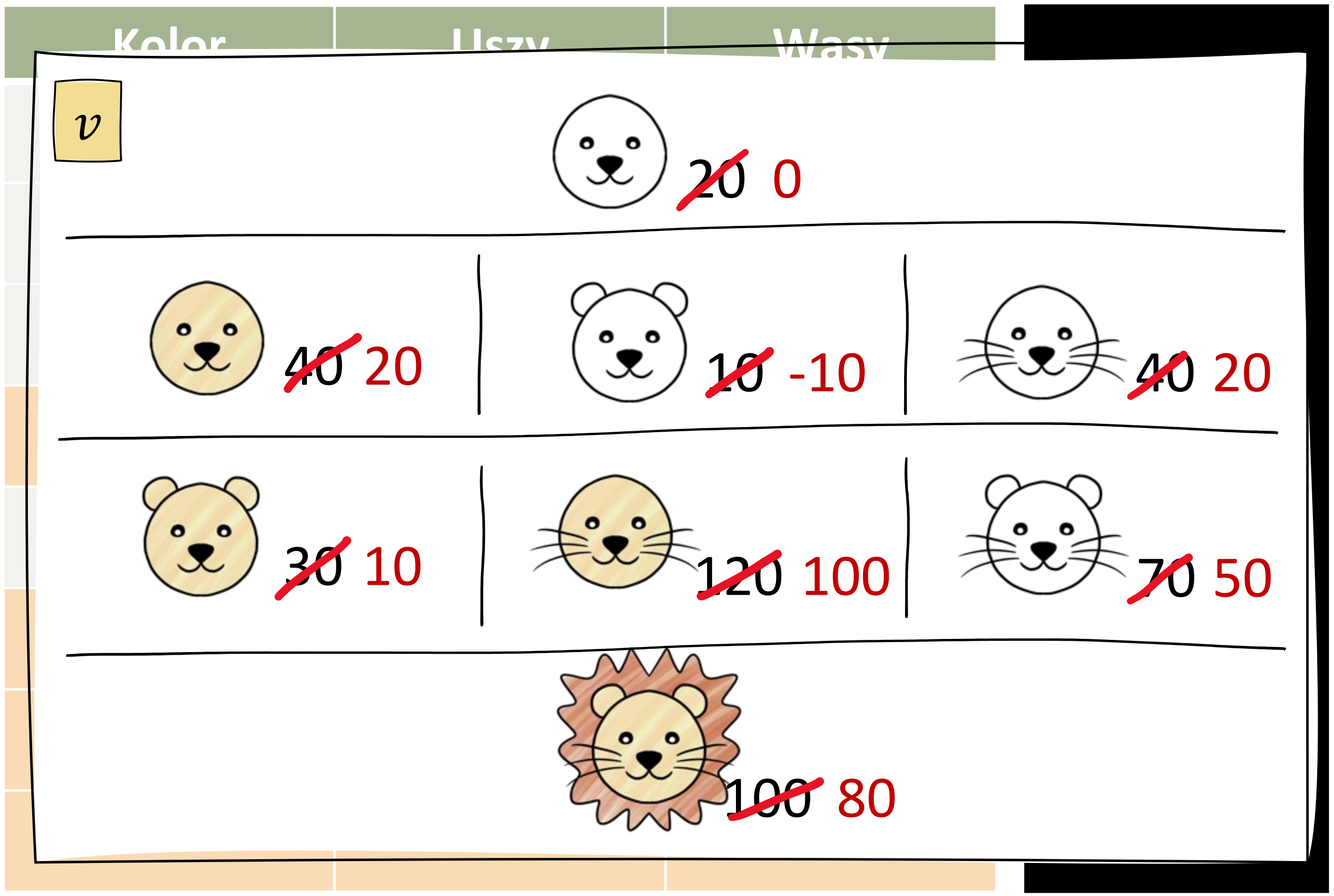
CZARNA SKRZYNKA

SHAP

Kolor	Uszy	Wąsy	Wynik
żółty	okrągłe	TAK	100
żółty	okrągłe	–	30
żółty	–	TAK	120
–	okrągłe	TAK	70
żółty	–	–	40
–	okrągłe	–	10
–	–	TAK	40
–	–	–	20

CZARNA SKRZYNKA

SHAP



Wynik
100
30
120
70
40
10
40
20

SHAP

Kolor	Uszy	Wąsy	[redacted]	Wynik
żółty	okrągłe	TAK		100
żółty	okrągłe	–		30
–	–	TAK		40
–	–	–		20

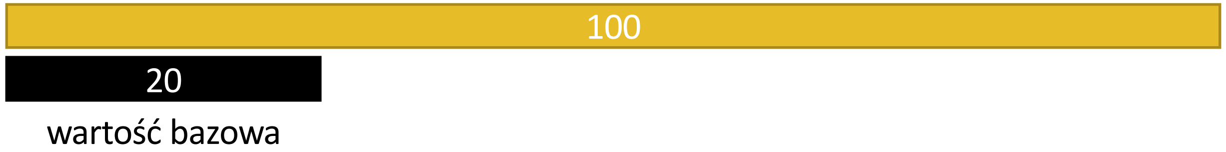
Czemu lew jest groźny?

100

SHAP

Kolor	Uszy	Wąsy	Wynik
żółty ...	okrągłe	TAK	100
żółty	okrągłe	–	30
–	–	TAK	40
–	–	–	20

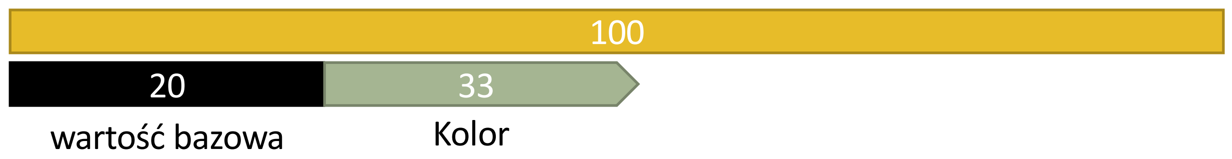
Czemu lew jest groźny?



SHAP

Kolor	Uszy	Wąsy	Wynik
żółty ...	okrągłe	TAK	100
żółty	okrągłe	–	30
–	–	TAK	40
–	–	–	20

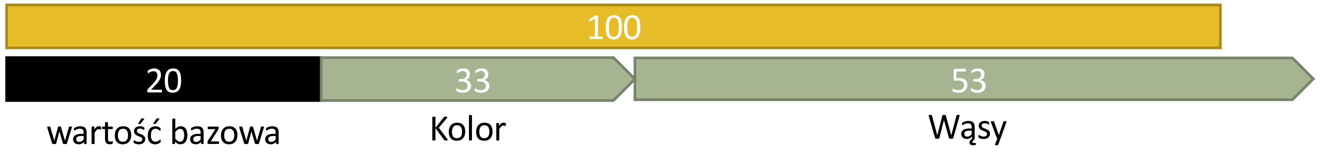
Czemu lew jest groźny?



SHAP

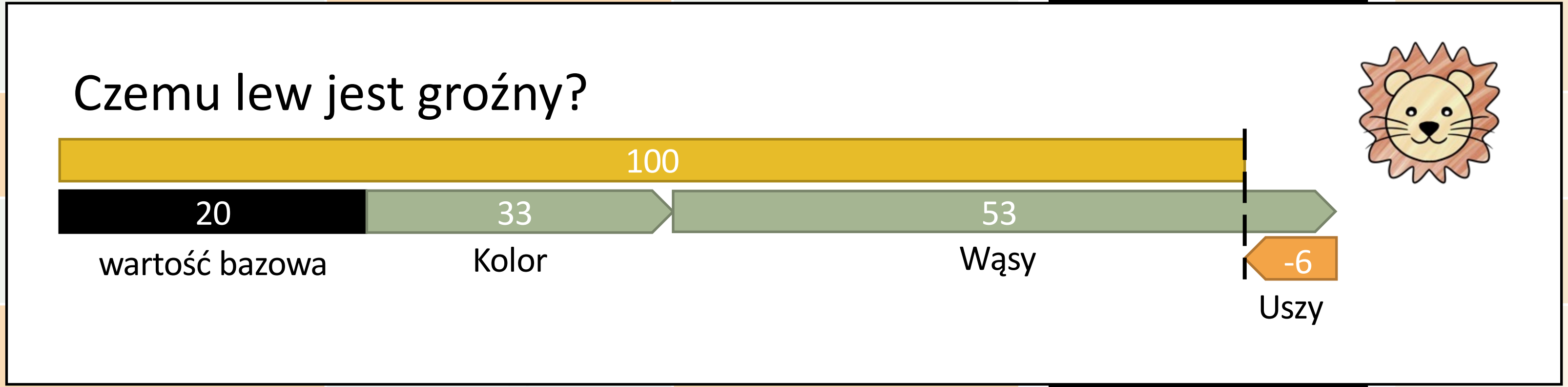
Kolor	Uszy	Wąsy	Wynik
żółty …	okrągłe	TAK	100
żółty	okrągłe	–	30
–	–	TAK	40
–	–	–	20

Czemu lew jest groźny?



SHAP

Kolor	Uszy	Wąsy	Wynik
żółty …	okrągłe	TAK	100
żółty	okrągłe	–	30
–	–	TAK	40
–	–	–	20



SHAP

Kolor	Uszy	Wąsy	Wynik
żółty	okrągłe	TAK	100
żółty	okrągłe	–	30
–	–	TAK	40
–	–	–	20

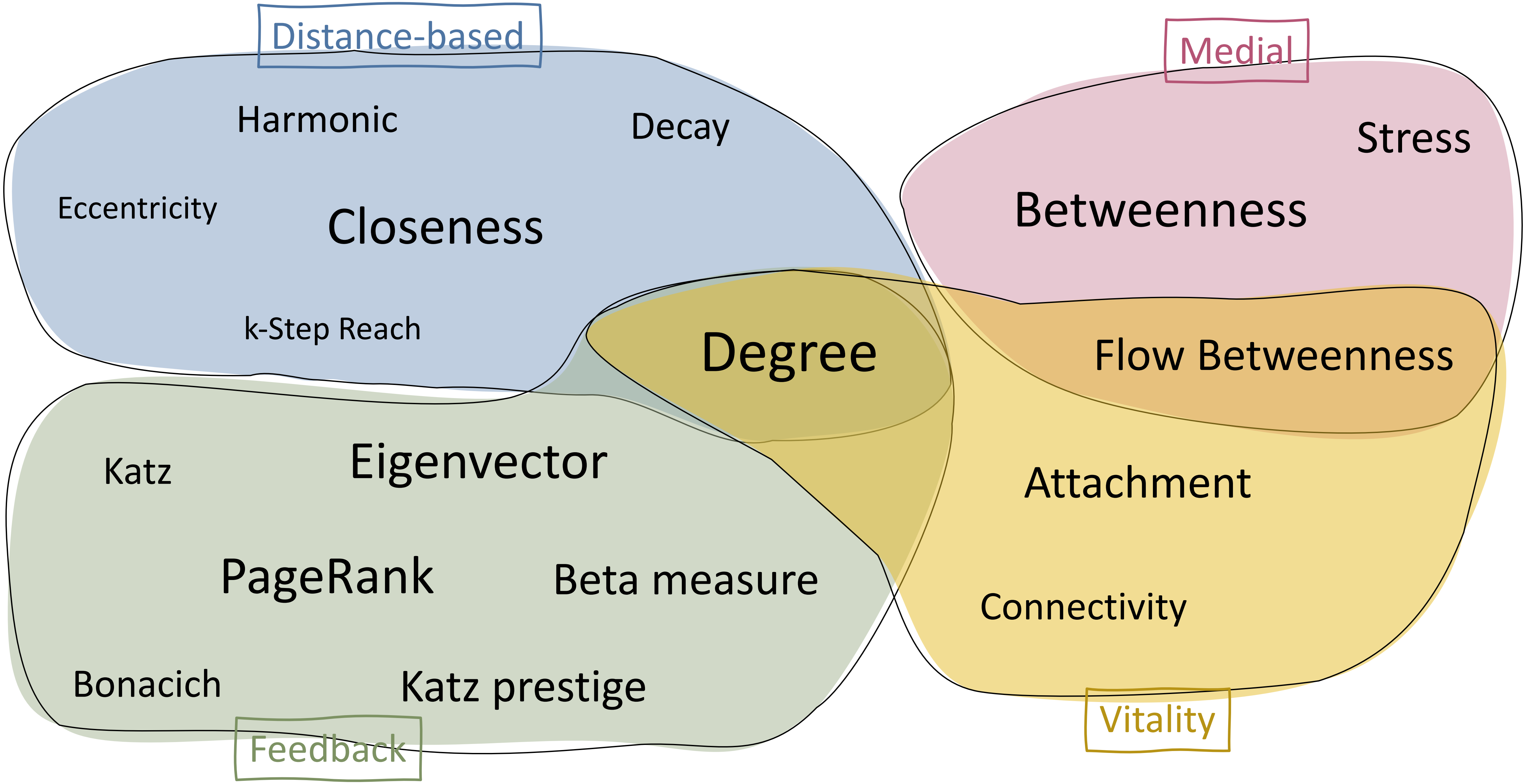
SHAP

!!! TEMAT !!!

Stworzenie biblioteki do oceny grup cech w algorytmach ML

Implementacja (głównie)

Vitality Indices



Which centrality measure we should use?

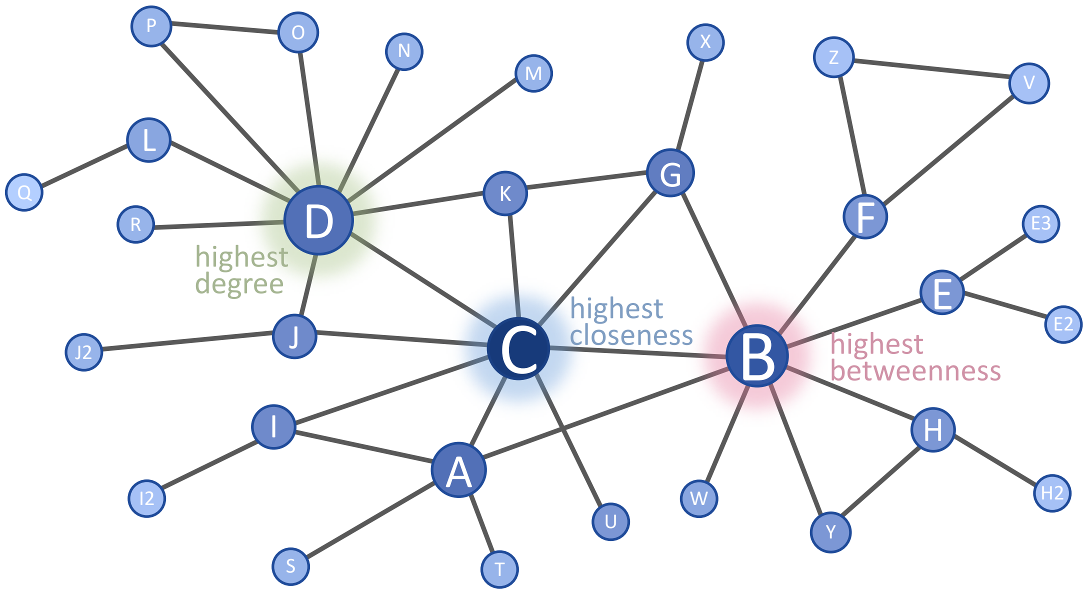
- Characterizing all centralities [Sabidussi 1966; Nieminen 1973]
- Testing satisfiability of simple properties [Boldi, Vigna 2014; Boldi, Luongo, Vigna 2017]
- Axiomatizations of Katz prestige [Palacios-Huerta, Volij 2004; Altman, Tennenholtz 2005] and of PageRank [Wąs, S. 2020]
- Axiomatizations of other feedback centralities [van den Brink, Gilles 2000; Dequiedt, Zenou 2014; Kitti 2016; Wąs, S. 2018]
- Axiomatizations of distance-based centralities [Garg 2009; S., Sosnowska 2018, S. 2023]
- All centrality measures process different informations about nodes in the same way [Bloch, Jackson, Tebaldi 2016]

Conclusions

There are many centrality measures...

Most standard centralities belong to one of four classes:

- Distance-based
- Medial
- Feedback
- Vitality



The choice depends on a setting/application at hand.

Thank you for listening!