

# Fairly Allocating Goods and (Terrible) Chores

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Hadi Hosseini, Agha Mammadov, Tomasz Was



IJCAI/2023 MACAO

# Fair Division under Lexicographic Preferences

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# Allocation of Indivisible Items

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N - agents  $\{1, \dots, n\}$

# Allocation of Indivisible Items

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N - agents {1, ..., n}

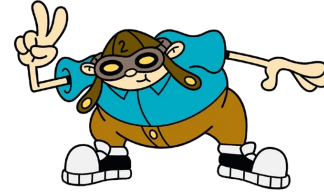


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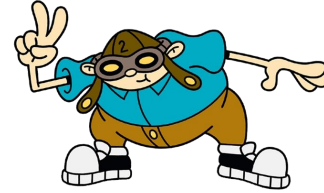
M - items  $\{o_1, \dots, o_m\}$



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$(\succeq_i)_{i \in N}$  - preferences over  $2^M$   
(weak orderings)





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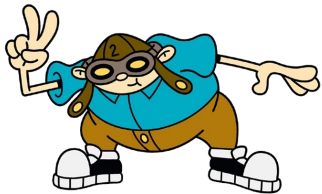
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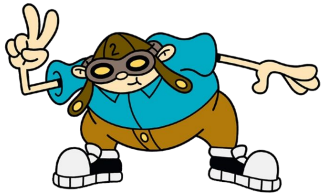
- a good

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(weak orderings)









$:$   $\left\{ \text{Christmas sweater} \right\} \prec \emptyset$

- a chore

# Additive Preferences

					
	6	7	0	1	0
	4	8	1	2	2
	4	5	3	1	-2

# Additive Preferences

						
	6	7	0	1	0	= 8
	4	8	1	2	2	
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# Lexicographic Preferences

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$\triangleright$



$\triangleright$



importance ordering,  $\triangleright_i$

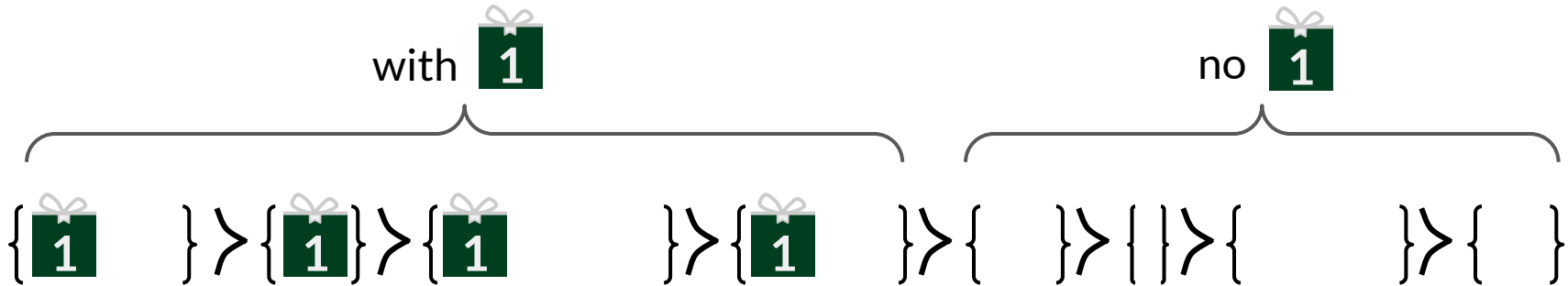
# Lexicographic Preferences

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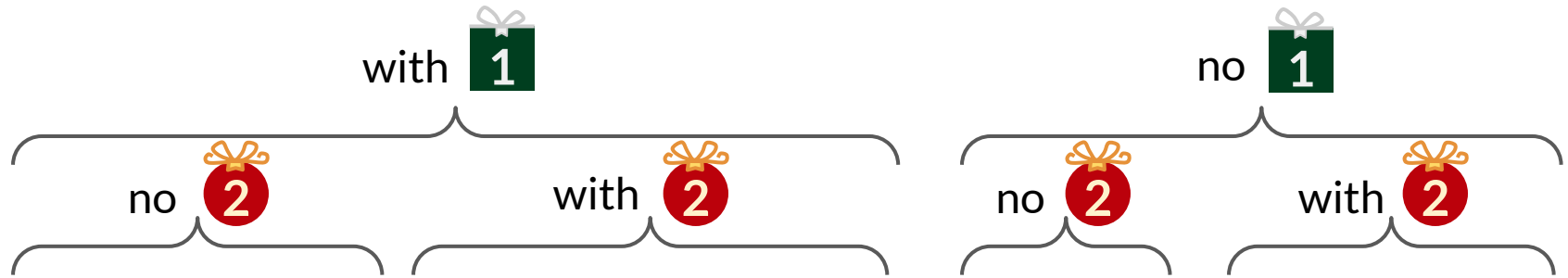




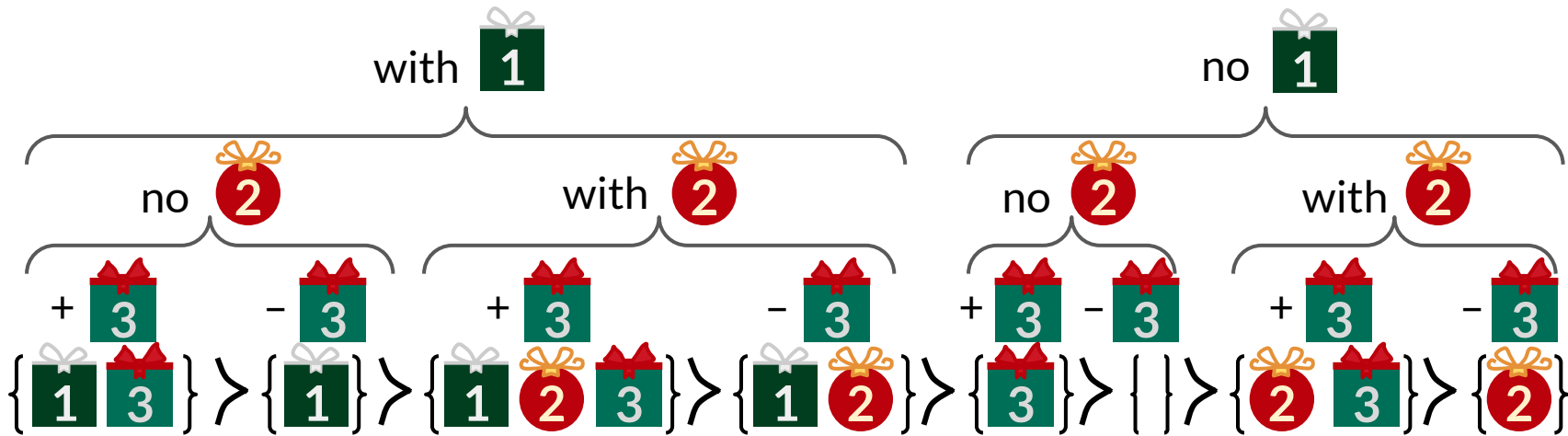
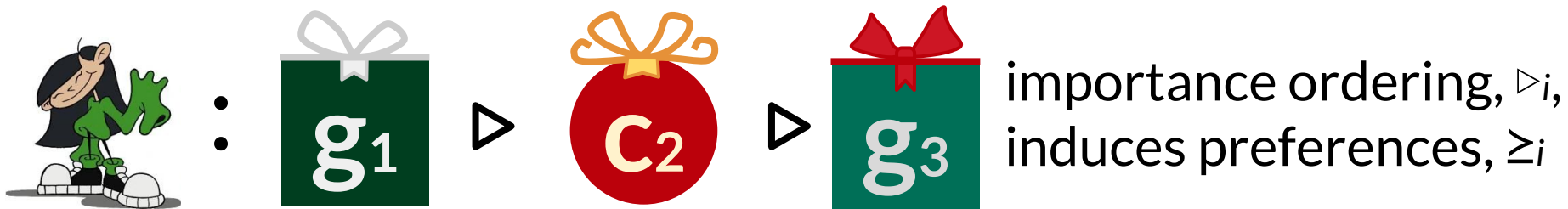
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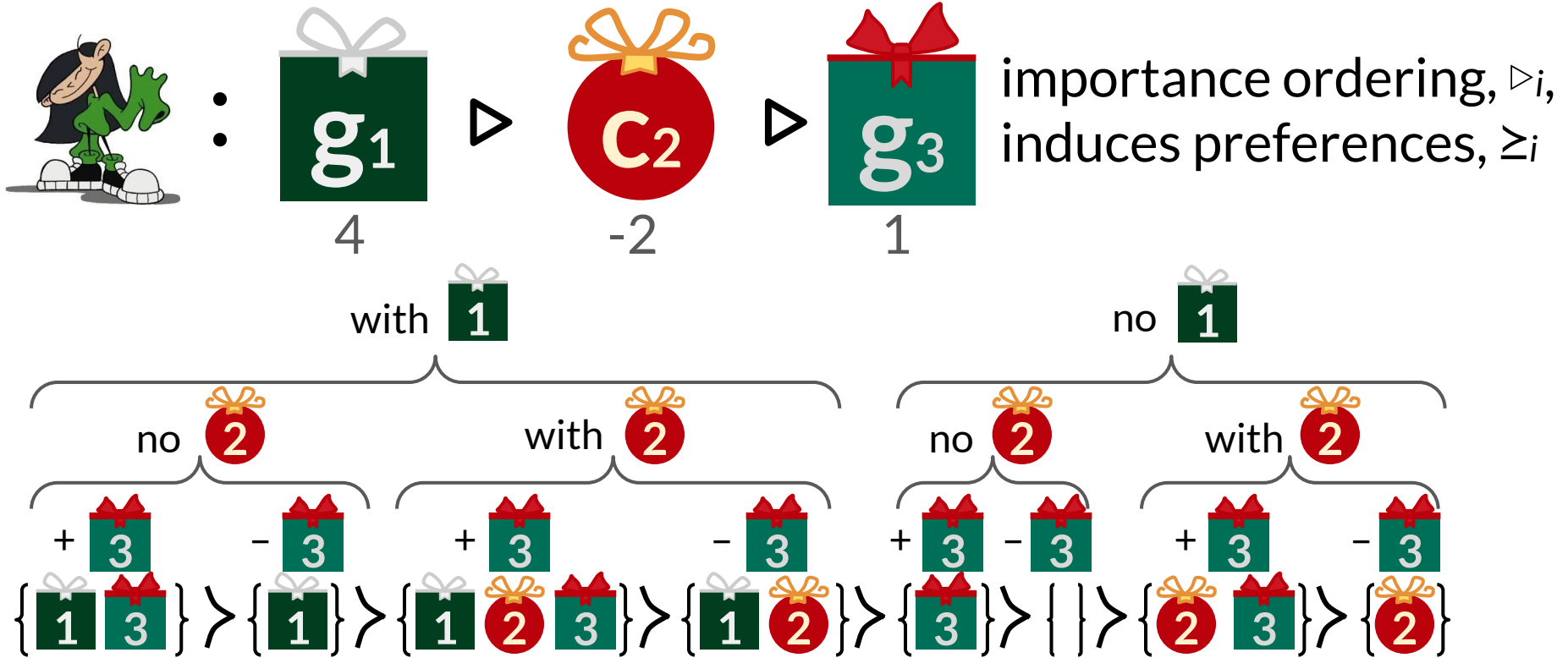
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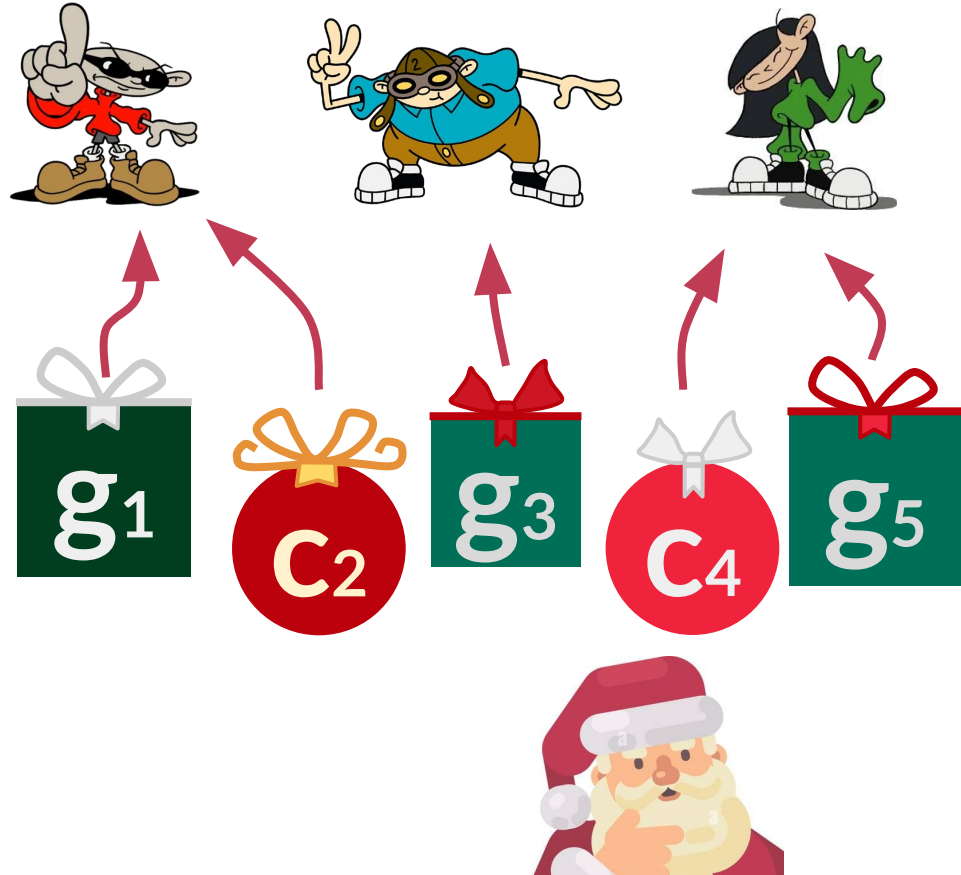
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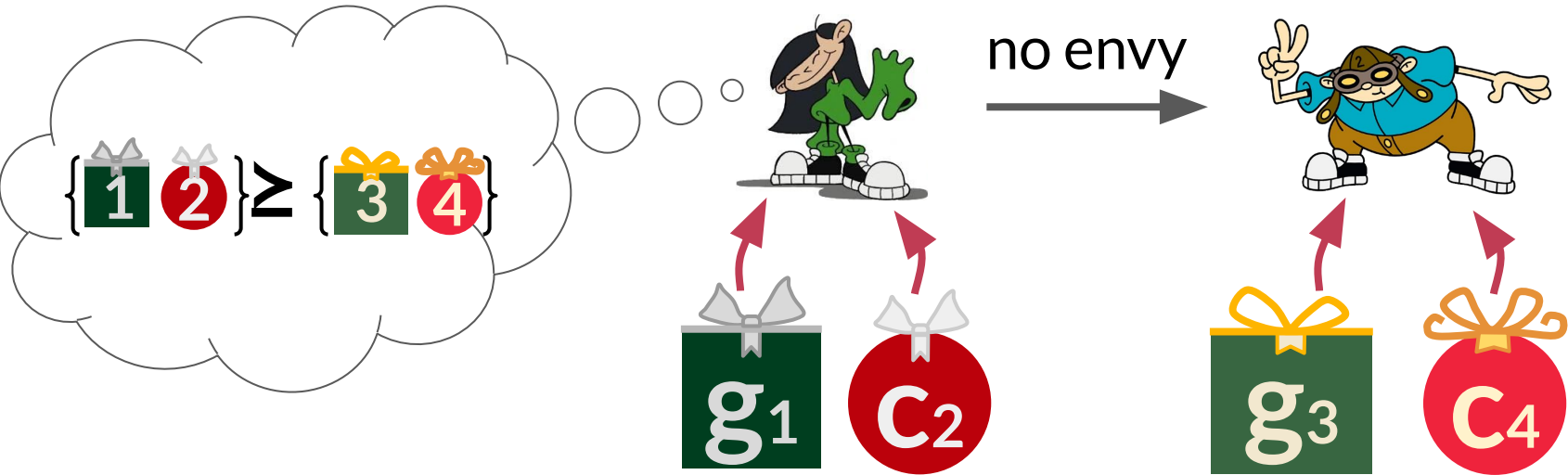
$(\succeq_i)_{i \in N}$  - preferences over  $2^M$   
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$A = (A_1, \dots, A_n)$  - allocation,  
partition of  $M$  with bundles  
assigned to agents



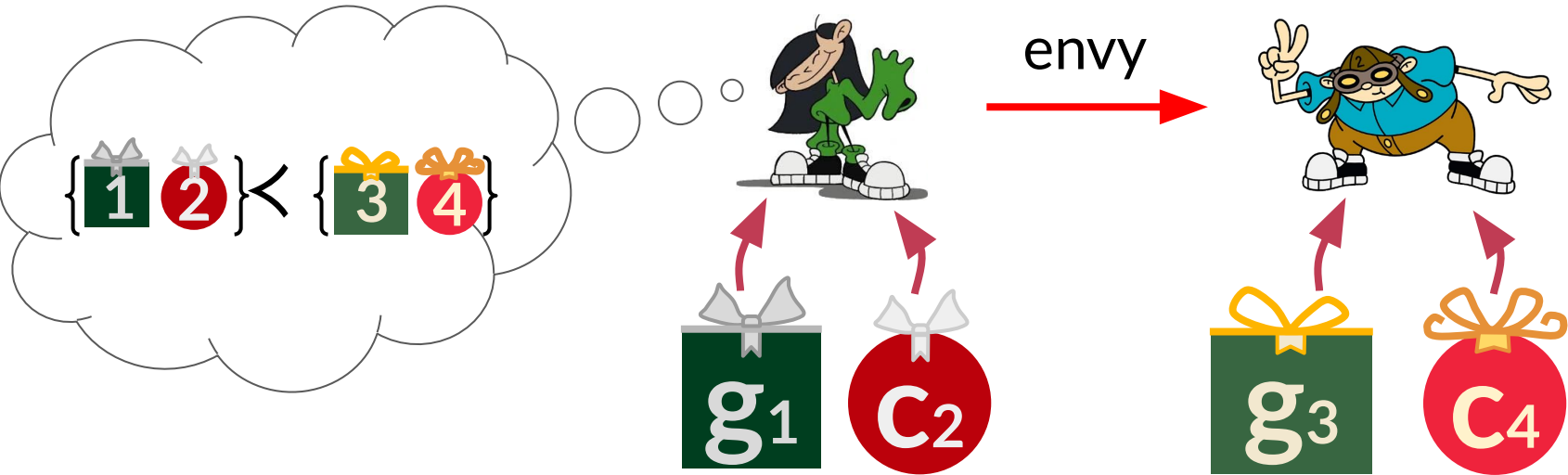
# Envy-Freeness (EF)

For every  $i, j \in N, A_i \succeq_i A_j$ .



# Envy-Freeness (EF)

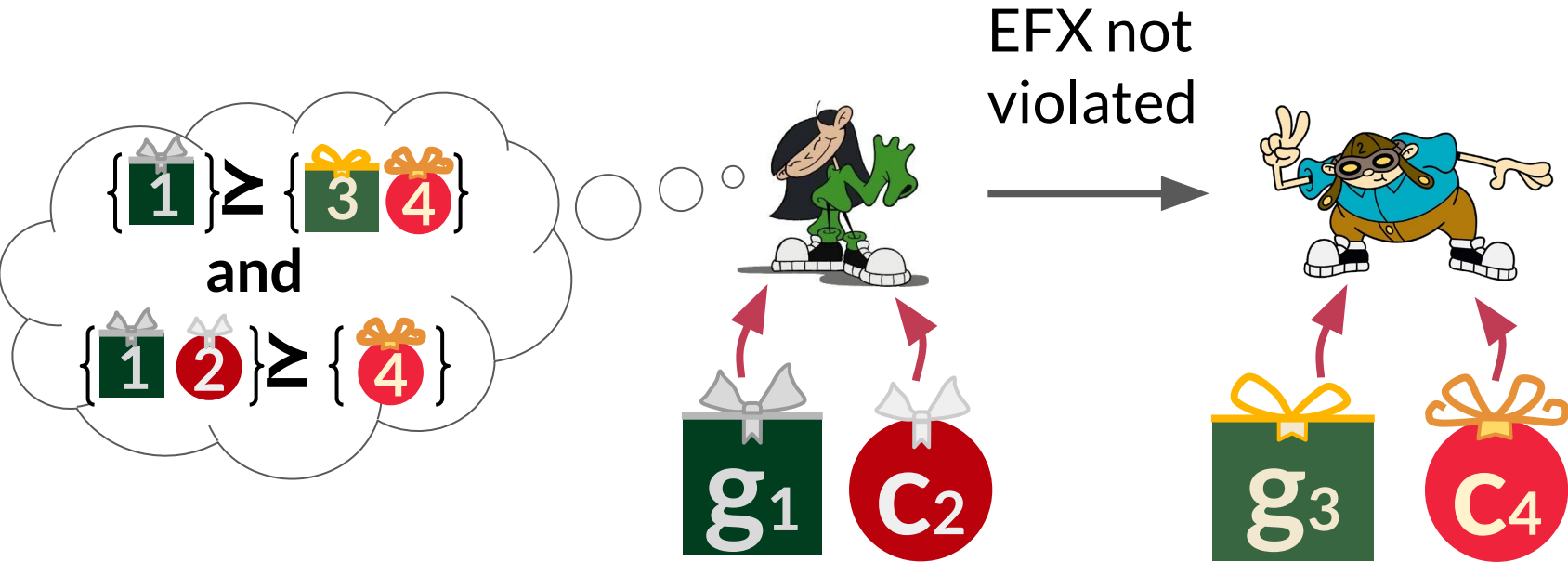
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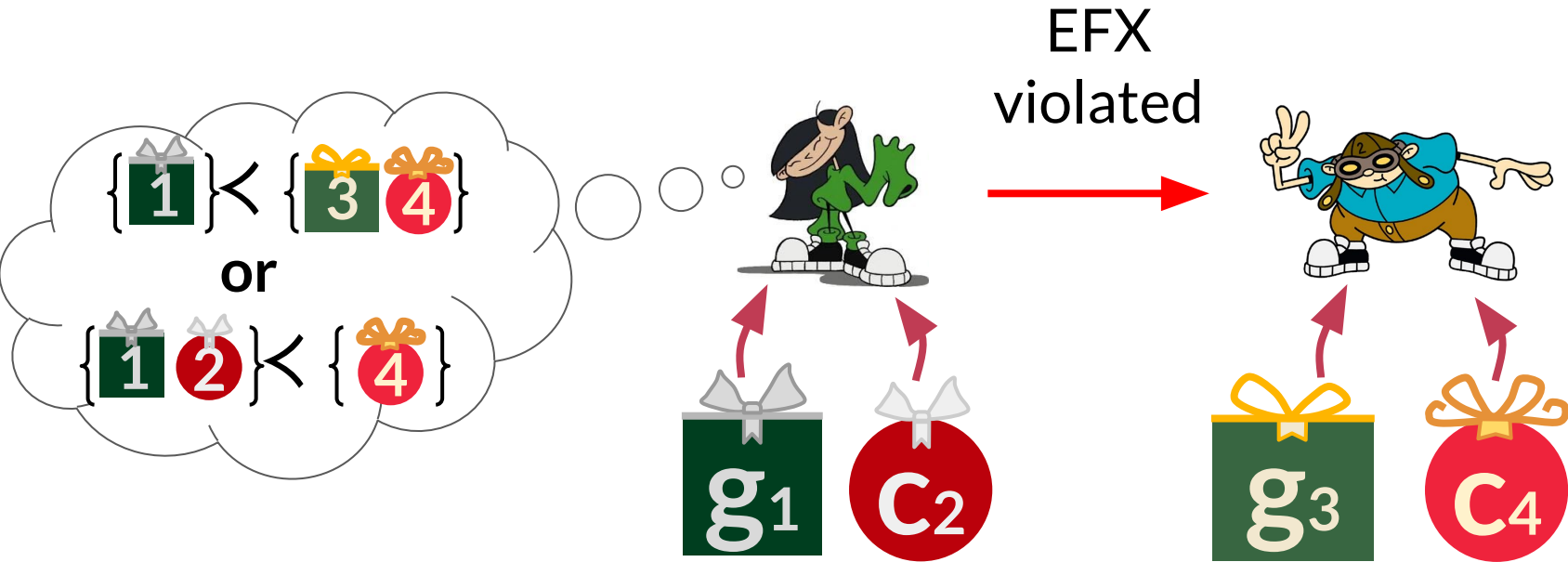
# Envy-Freeness up to Any Item (EFX)

For every  $i, j \in N$ , every chore  $c \in A_i$ , and every good  $g \in A_j$ , it holds that  $A_i \setminus \{c\} \succeq_i A_j$  and  $A_i \succeq_i A_j \setminus \{g\}$ .



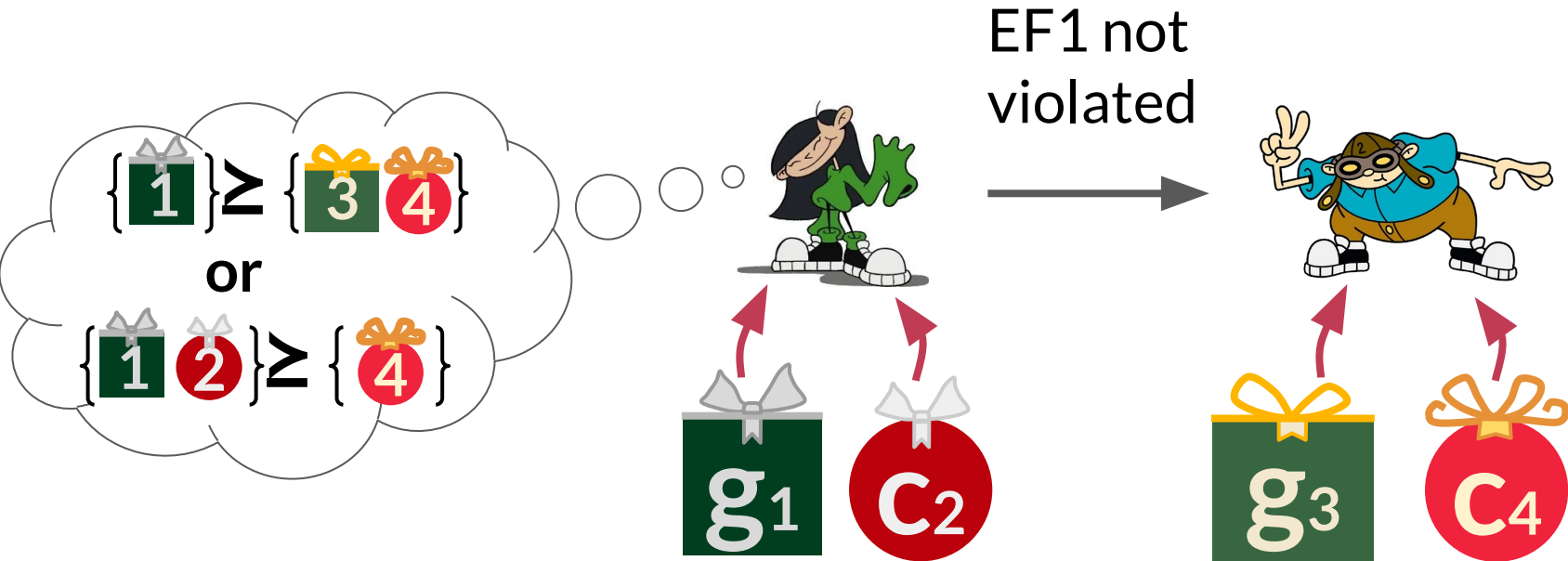
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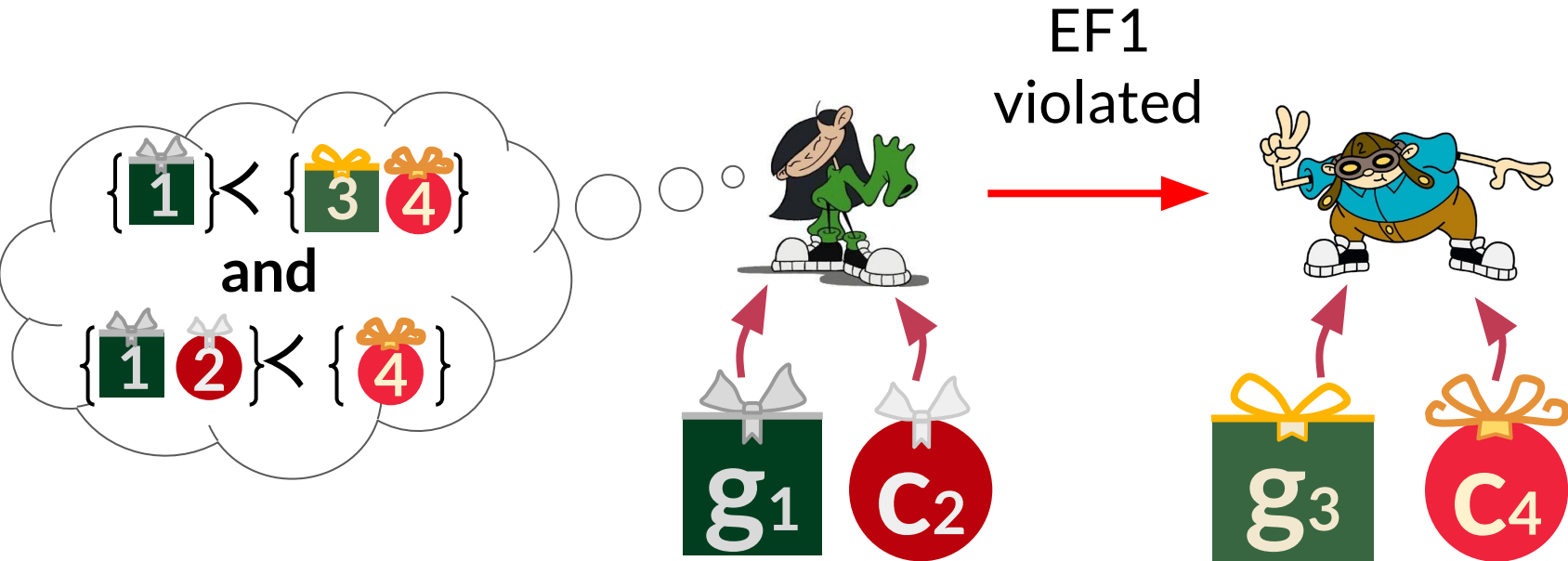
# Envy-Freeness up to One Item (EF1)

For every  $i, j \in N$ , there exists a chore  $c \in A_i$ , or a good  $g \in A_j$ , such that  $A_i \setminus \{c\} \succeq_i A_j$  or  $A_i \succeq_i A_j \setminus \{g\}$ .



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# MaxiMin Share (MMS)

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MaxiMin Share threshold of agent  $i \in N$  is:

$$MMS_i = \max_{A=(A_1, \dots, A_n)} \min_{j \in [n]} A_j.$$

Allocation  $A$  is MMS if for every  $i \in N$ , it holds that:

$$A_i \geq_i MMS_i$$

# Pareto Optimality (PO)

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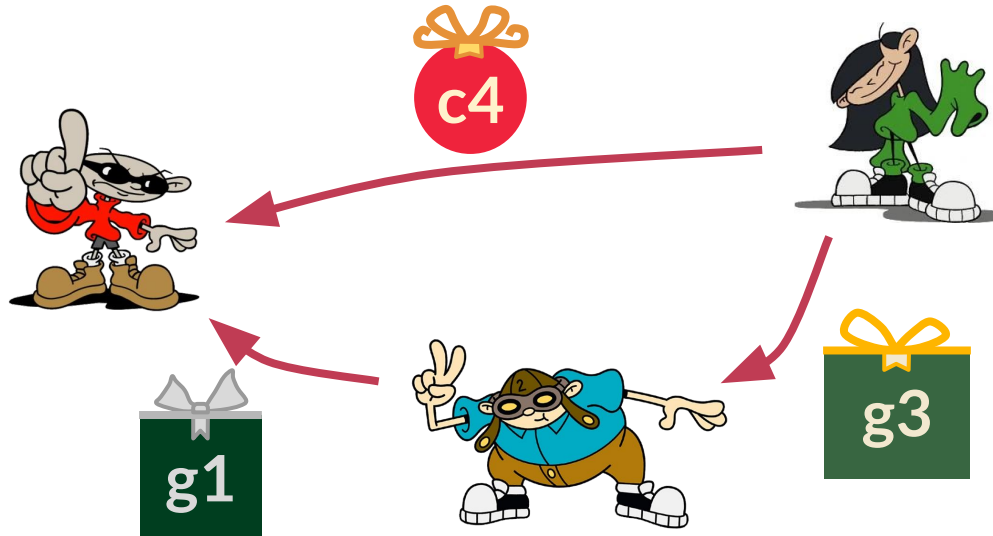
An allocation  $A$  is PO if there is no allocation  $B$  such that  $B_i \succeq_i A_i$  for every  $i \in N$  and  $B_i \succ_i A_i$  for some  $i \in N$ .

# Pareto Optimality (PO)

An allocation  $A$  is PO if there is no allocation  $B$  such that

$B_i \succeq_i A_i$  for every  $i \in N$  and  $B_i \succ_i A_i$  for some  $i \in N$ .

or, for lexicographic preferences, if there is no improving cycle

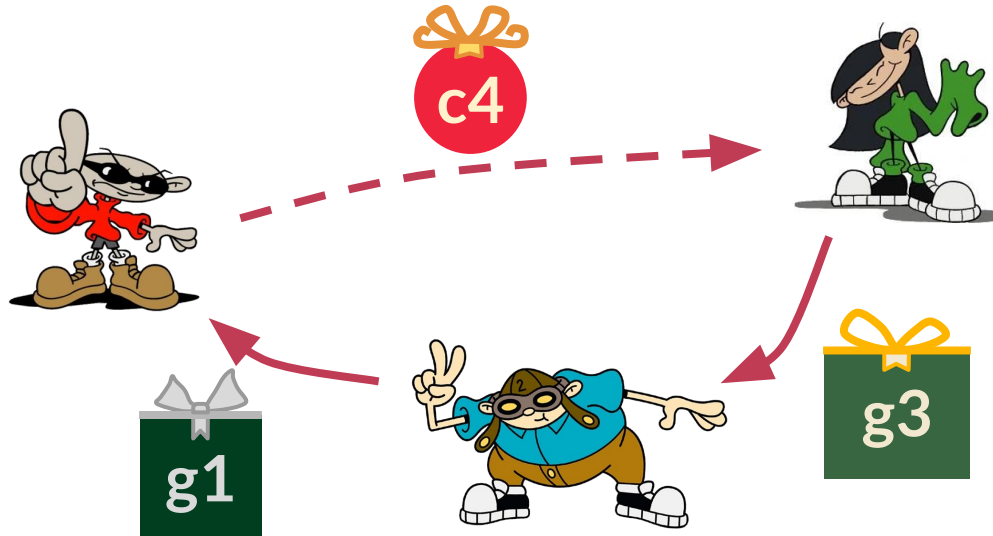


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or, for lexicographic preferences, if there is no improving cycle





# Additive Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
EFX	?	?	?
EFX + PO	?	?	?
EF1	 <span style="color: green; font-weight: bold; font-size: 1.2em;">P</span> [Lipton et al., 2004]	 <span style="color: green; font-weight: bold; font-size: 1.2em;">P</span> [Aziz et al., 2022]	 <span style="color: green; font-weight: bold; font-size: 1.2em;">P</span> [Aziz et al., 2022]
EF1 + PO	 <span style="color: orange; font-weight: bold; font-size: 1.2em;">pseudo-P</span> [Caragiannis et al., 2019]	?	?
MMS	 <span style="color: blue; font-weight: bold; font-size: 1.2em;">deciding?</span> [Procaccia & Wang, 2014]	 <span style="color: blue; font-weight: bold; font-size: 1.2em;">deciding?</span> [Aziz et al., 2017]	 <span style="color: blue; font-weight: bold; font-size: 1.2em;">deciding?</span> [Procaccia & Wang, 2014]
MMS + PO	 <span style="color: blue; font-weight: bold; font-size: 1.2em;">deciding?</span> [Procaccia & Wang, 2014]	 <span style="color: blue; font-weight: bold; font-size: 1.2em;">deciding?</span> [Aziz et al., 2017]	 <span style="color: blue; font-weight: bold; font-size: 1.2em;">deciding?</span> [Procaccia & Wang, 2014]

# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
EFX	?	?	?
EFX + PO	?	?	?
EF1	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
EF1 + PO	✓ pseudo-P [Caragiannis et al., 2019]	?	?
MMS	?	?	?
MMS + PO	?	?	?

# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
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EFX + PO	?	?	?
EF1	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
EF1 + PO	✓ pseudo-P [Caragiannis et al., 2019]	?	?
MMS	?	?	?
MMS + PO	?	?	?

# EFX for Lexicographic Goods



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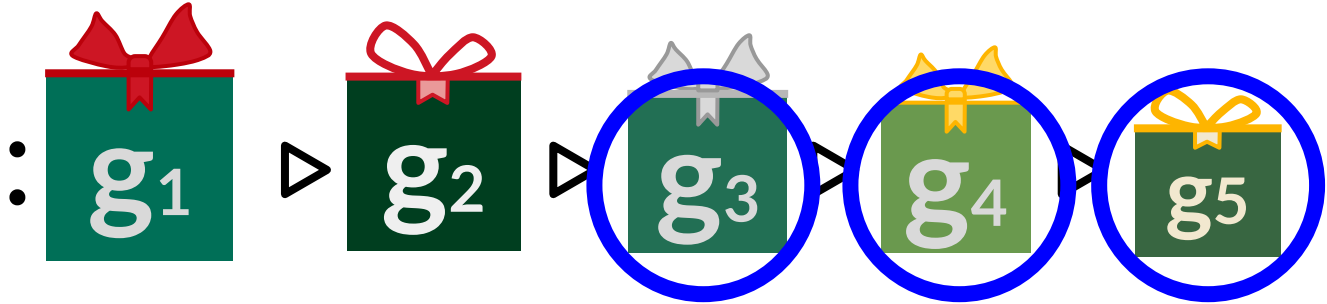
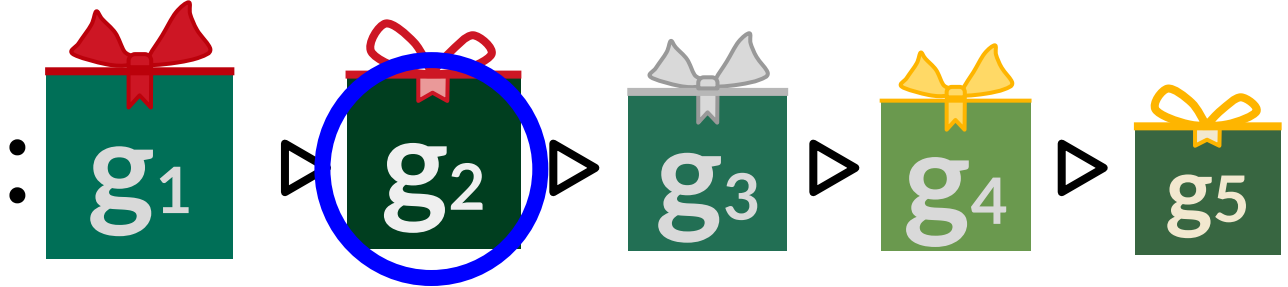


# EFX for Lexicographic Goods



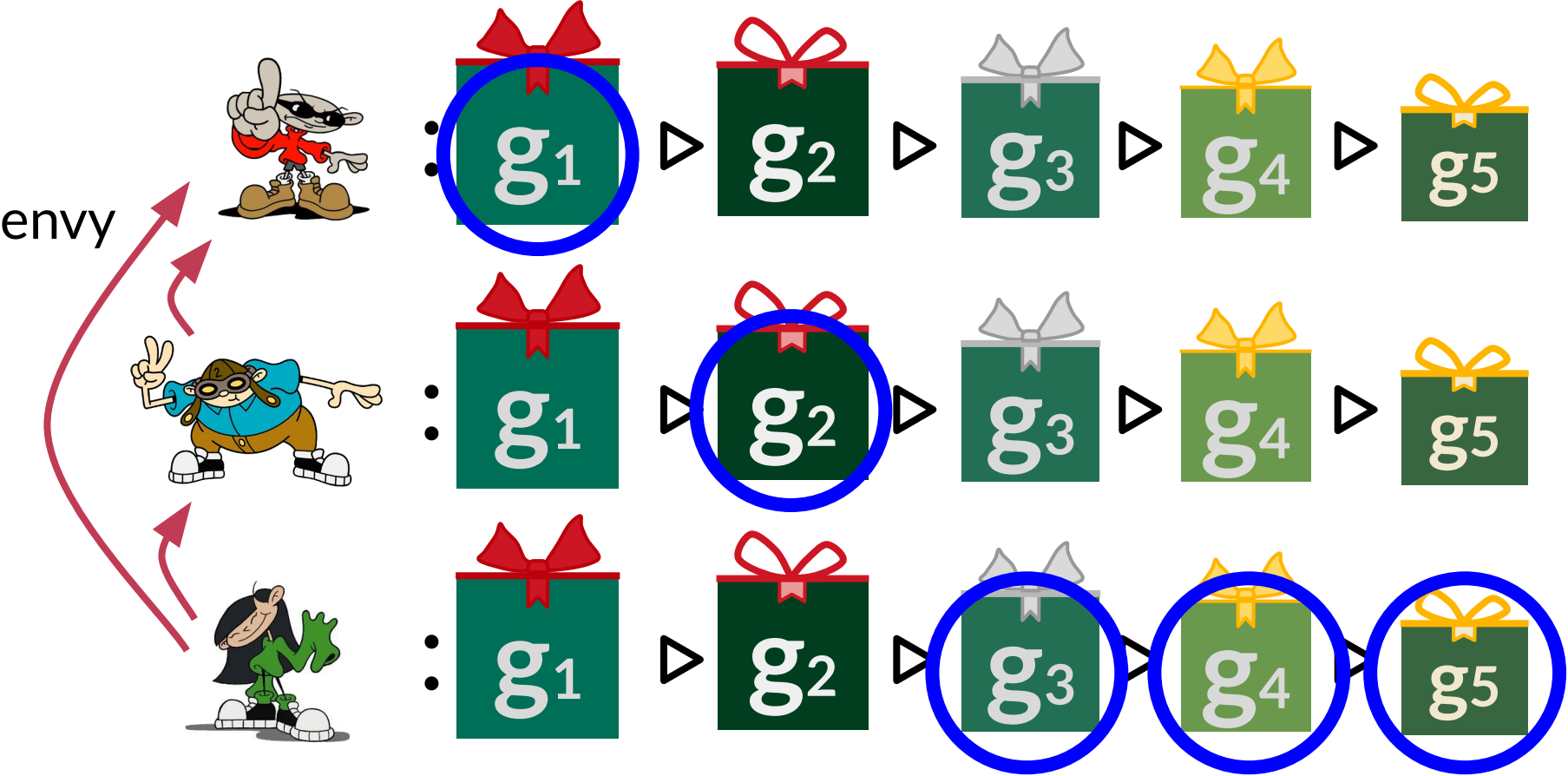
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no envy

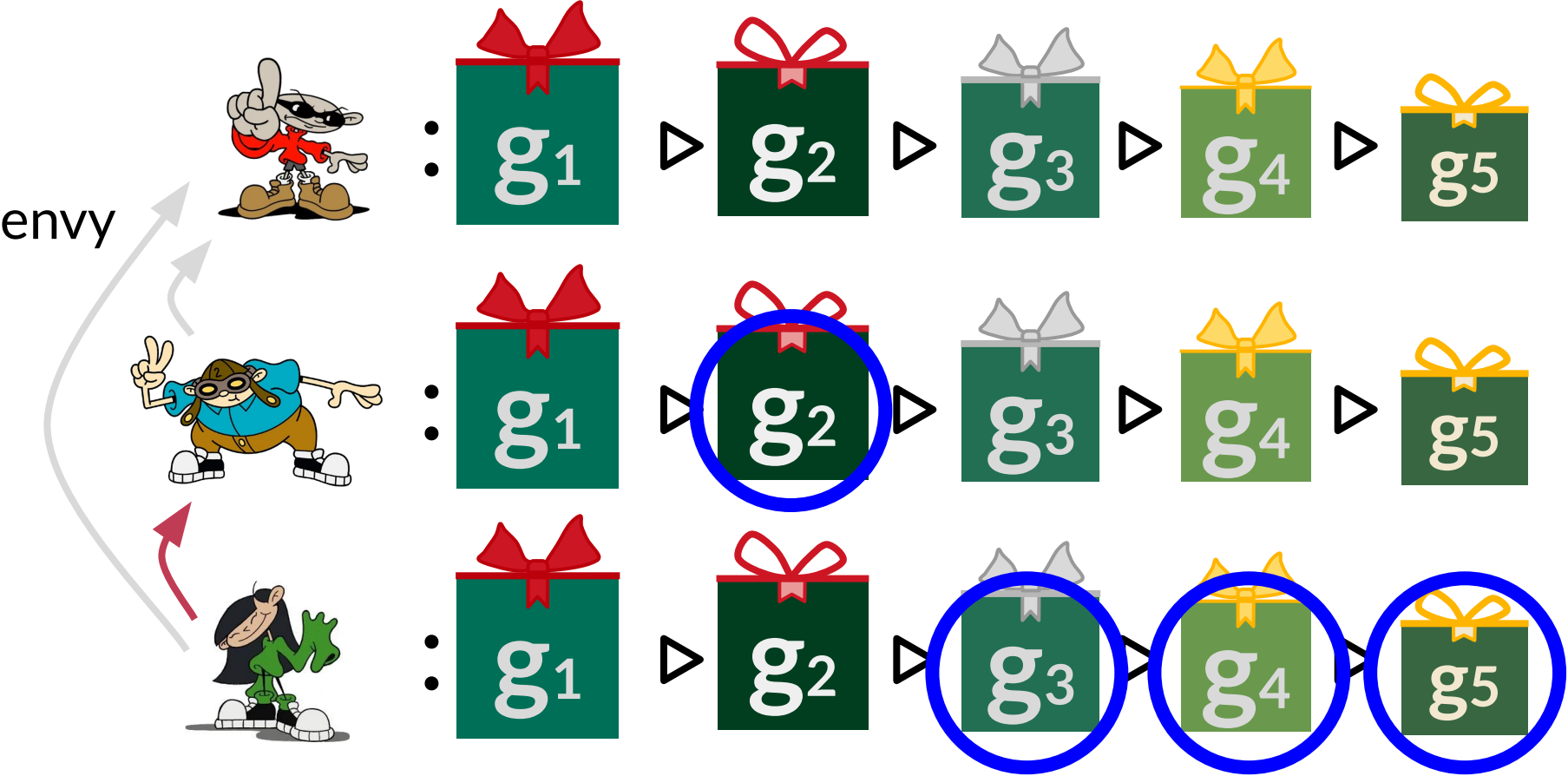




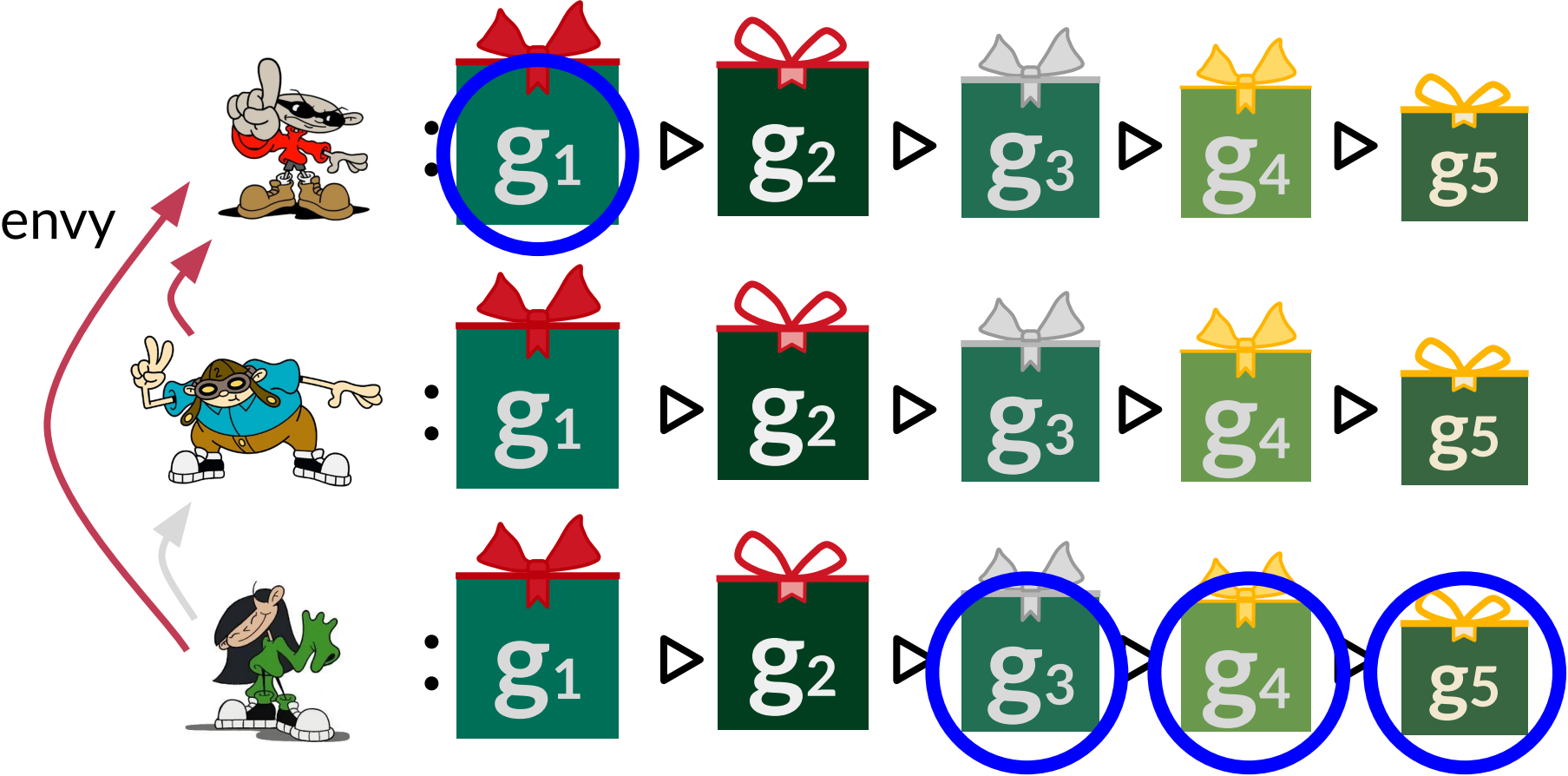
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# Lexicographic Preferences State of Knowledge

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EFX + PO	?	?	?
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EF1 + PO	✓ pseudo-P [Caragiannis et al., 2019]	?	?
MMS	?	?	?
MMS + PO	?	?	?

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EF1 + PO	✓ pseudo-P [Caragiannis et al., 2019]	?	?
MMS	?	?	?
MMS + PO	?	?	?

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EF1	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
EF1 + PO	✓ P [Hosseini et al., 2021]	?	?
MMS	?	?	?
MMS + PO	?	?	?

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EF1 + PO	✓ P [Hosseini et al., 2021]	?	?
MMS	?	?	?
MMS + PO	?	?	?



# MMS for lexicographic preferences

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$$\text{MMS}_{\text{char}} = \left\{ \begin{array}{c} \text{yellow bow} \\ \text{red circle} \\ 1 \end{array}, \begin{array}{c} \text{red bow} \\ \text{green box} \\ 1 \end{array}, \begin{array}{c} \text{gray bow} \\ \text{green box} \\ 2 \end{array}, \dots, \begin{array}{c} \text{yellow bow} \\ \text{green box} \\ m \end{array} \right\}$$

# MMS for lexicographic preferences

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$$\text{MMS}_{\text{char}} = \{ \text{gift}_1, \text{gift}_1, \text{gift}_2, \dots, \text{gift}_m \}$$



$$\text{MMS}_{\text{char}} = \{ \text{gift}_n, \text{gift}_{n+1}, \dots, \text{gift}_m \}$$

# Lexicographic Preferences State of Knowledge

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EF1	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
EF1 + PO	✓ P [Hosseini et al., 2021]	?	?
MMS	?	?	?
MMS + PO	?	?	?

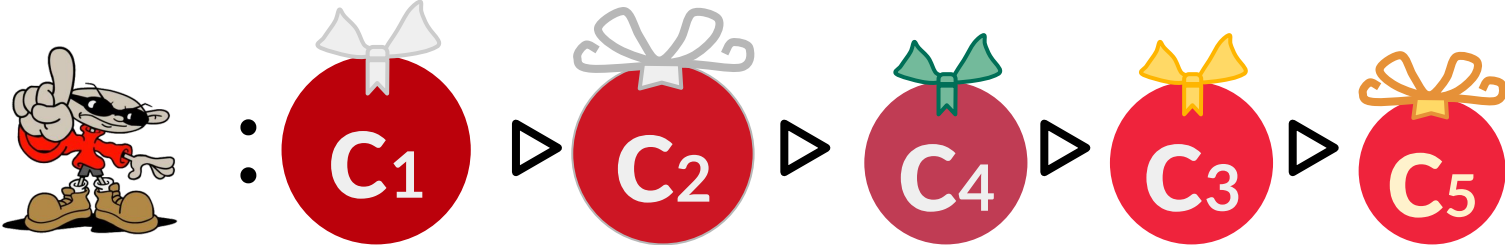
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EF1 + PO	✓ P [Hosseini et al., 2021]	?	?
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MMS + PO	✓ P [Hosseini et al., 2021]	?	?

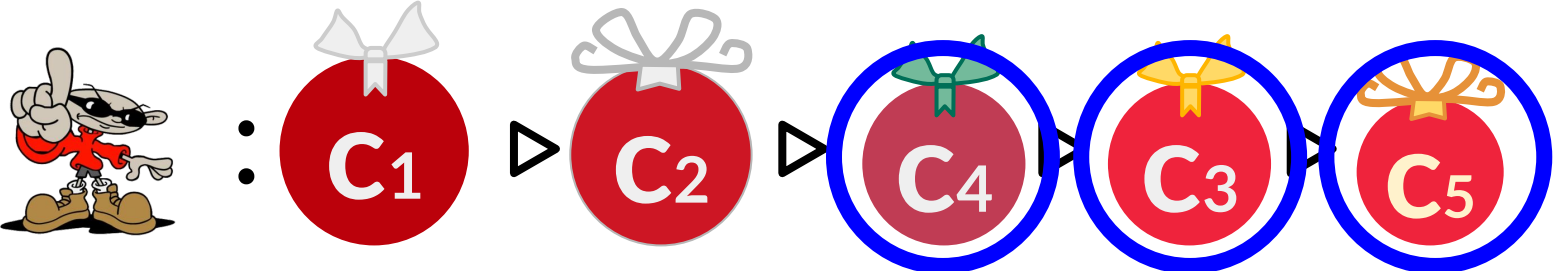
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EF1	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
EF1 + PO	✓ P [Hosseini et al., 2021]	?	?
MMS	✓ P [Hosseini et al., 2021]	?	?
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# EFX for Lexicographic Chores

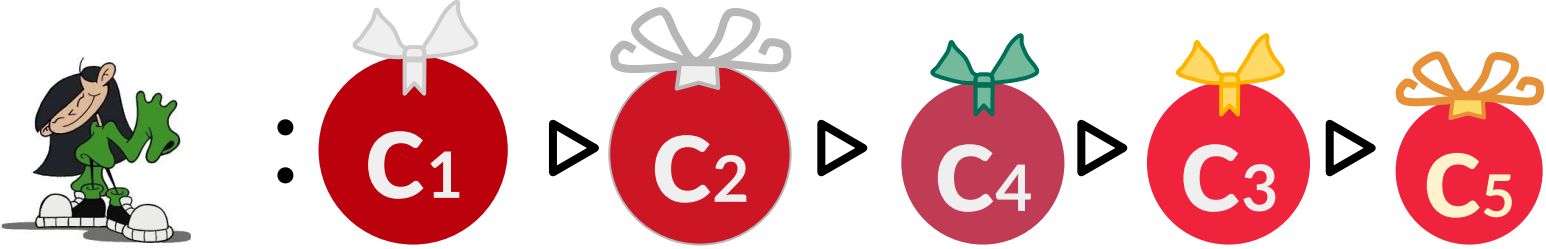
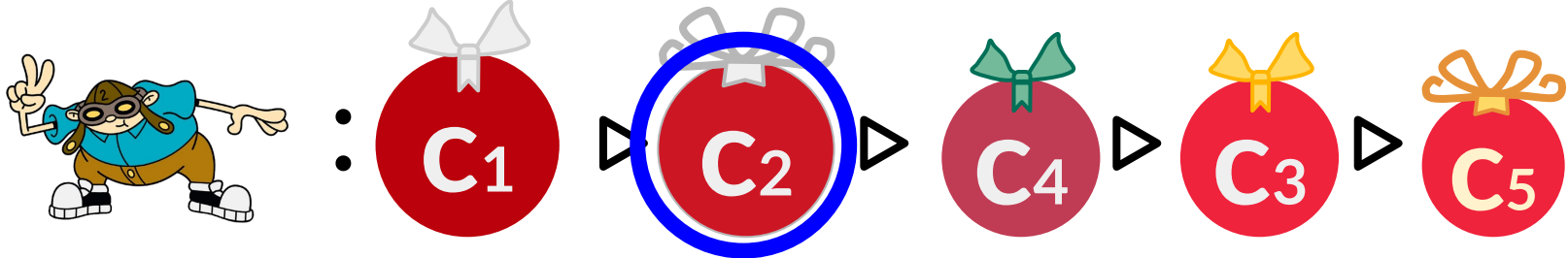
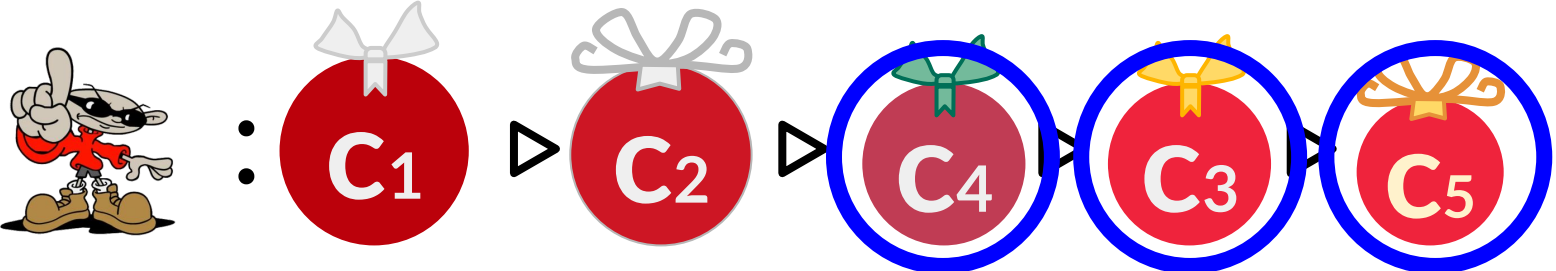


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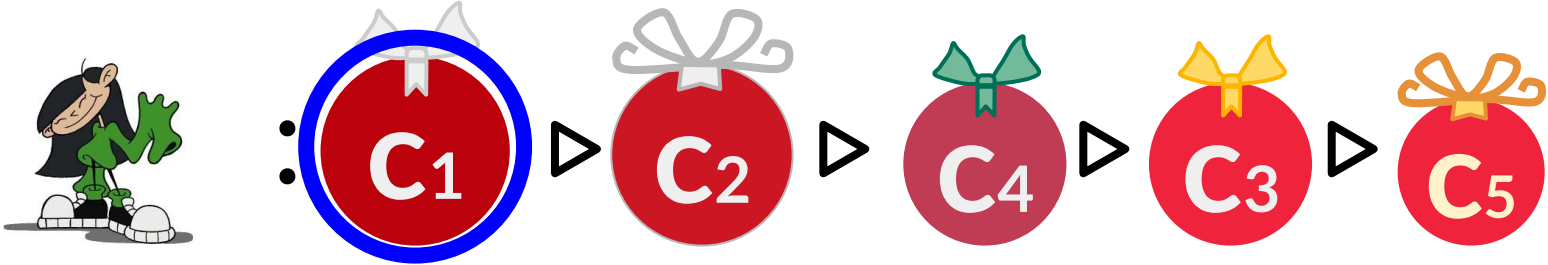
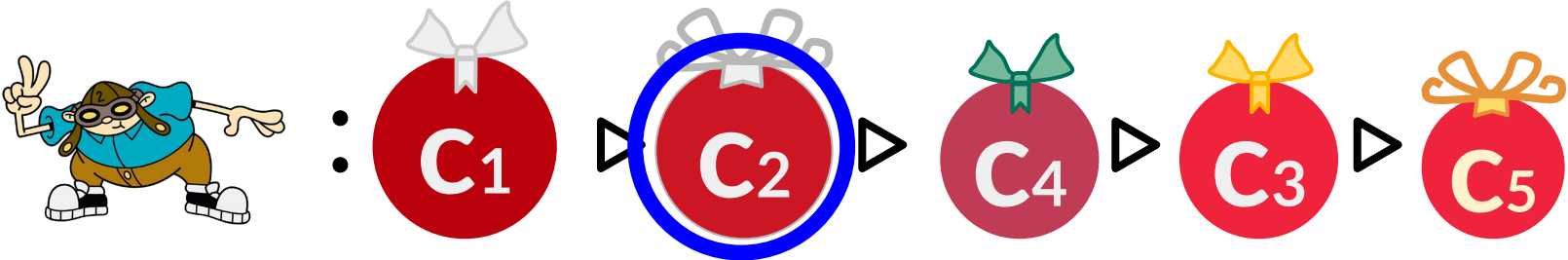
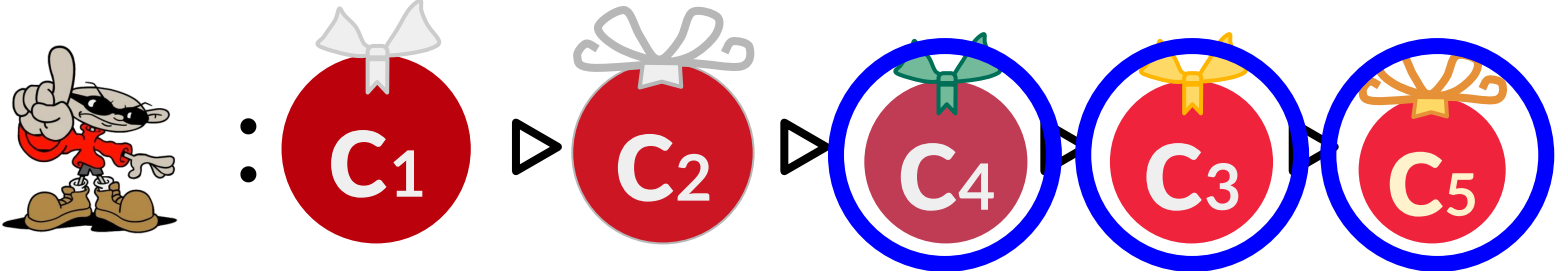




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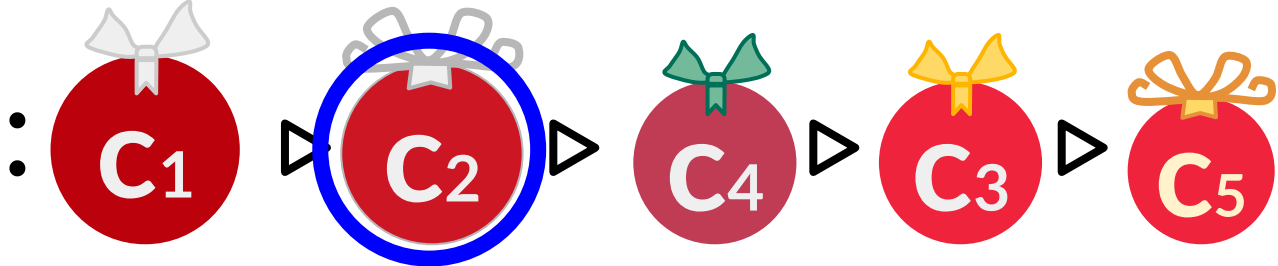
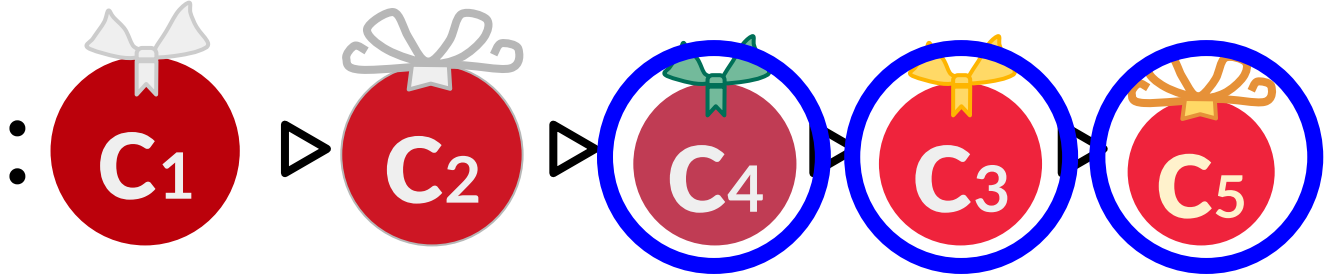


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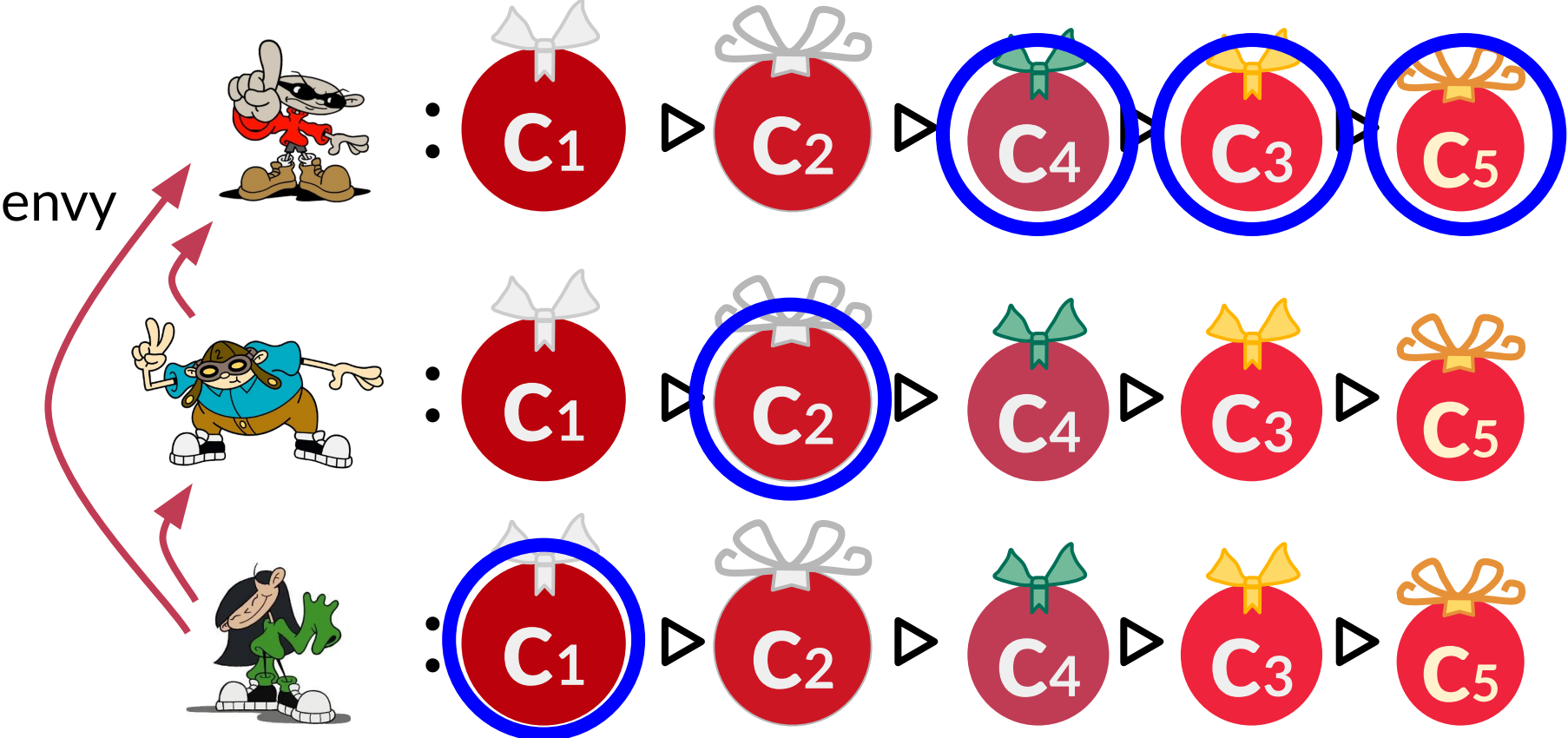


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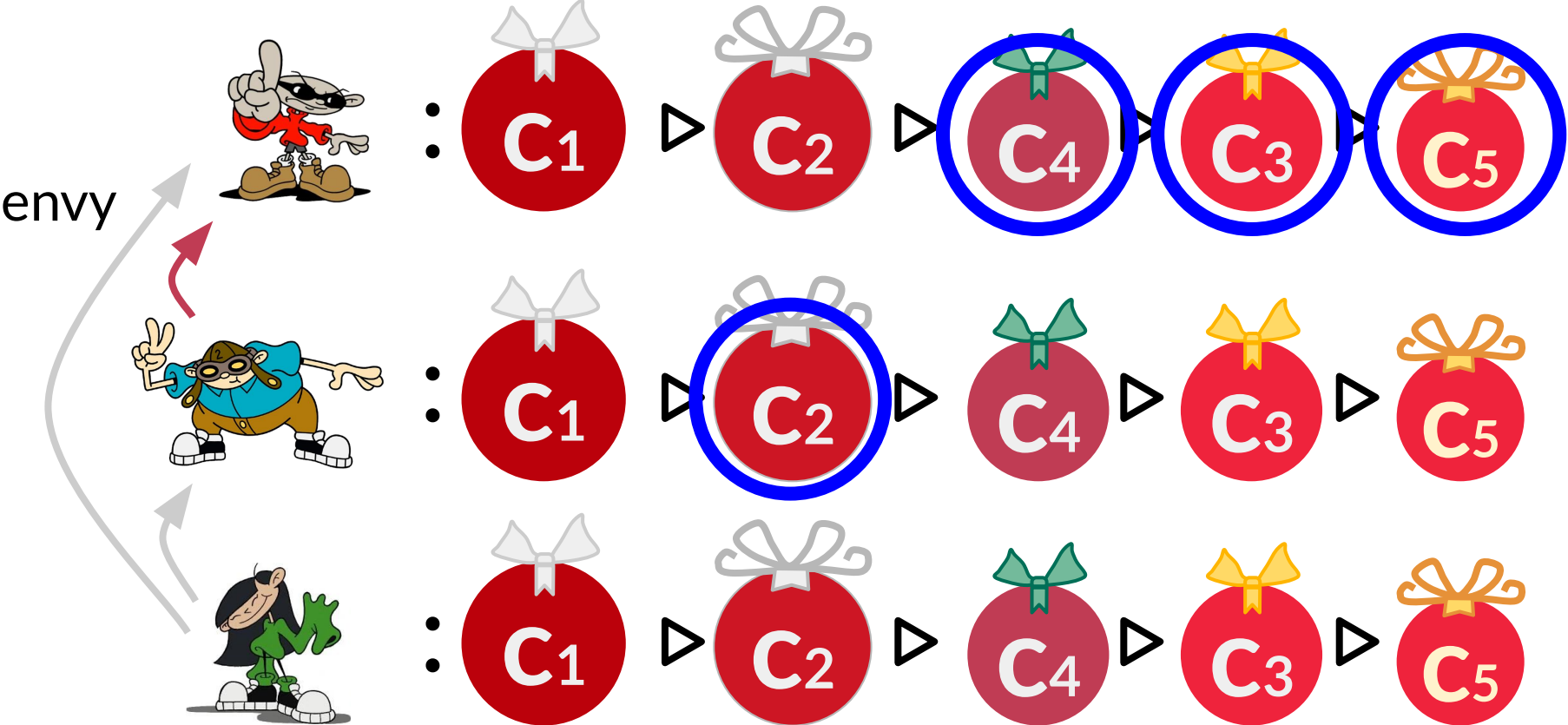
no envy



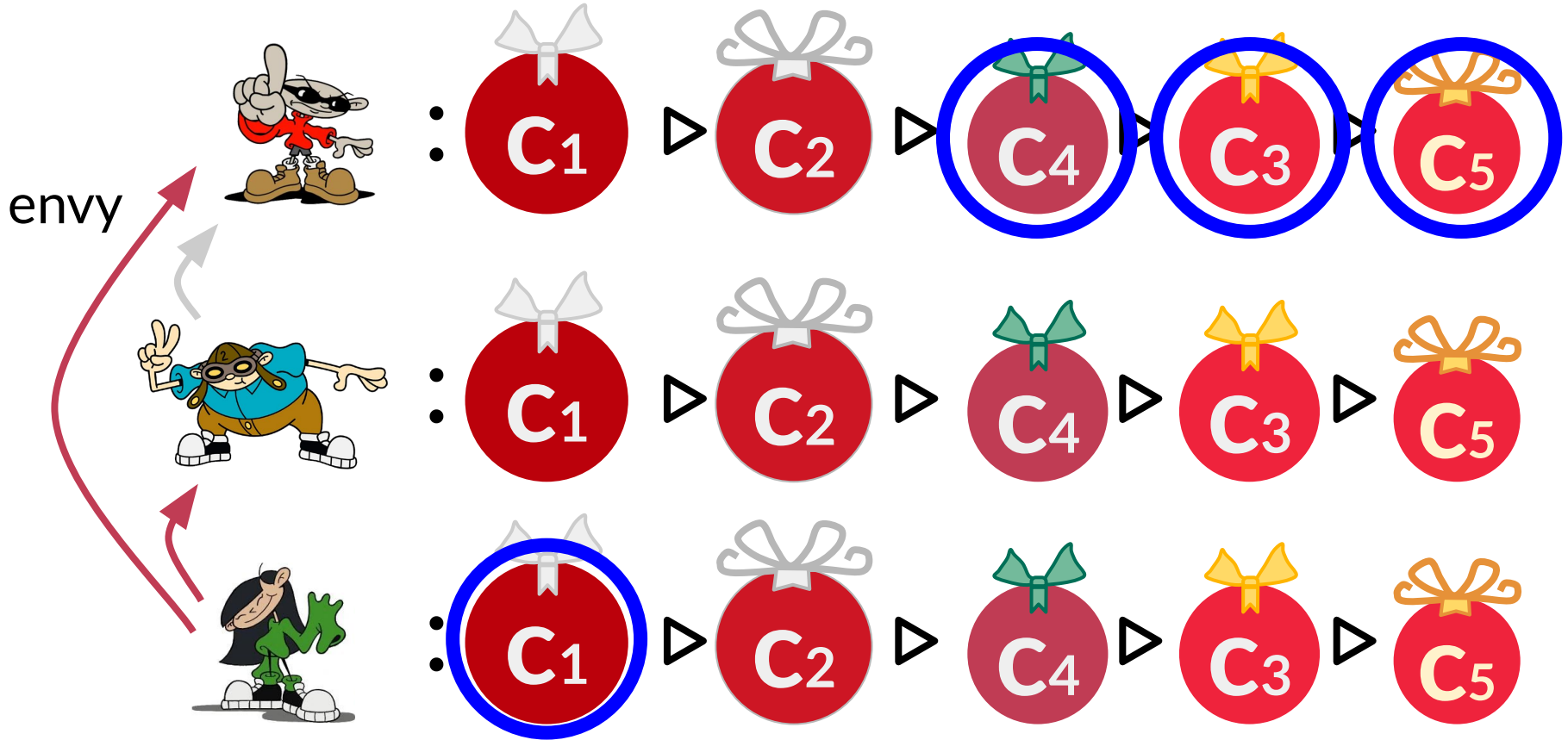
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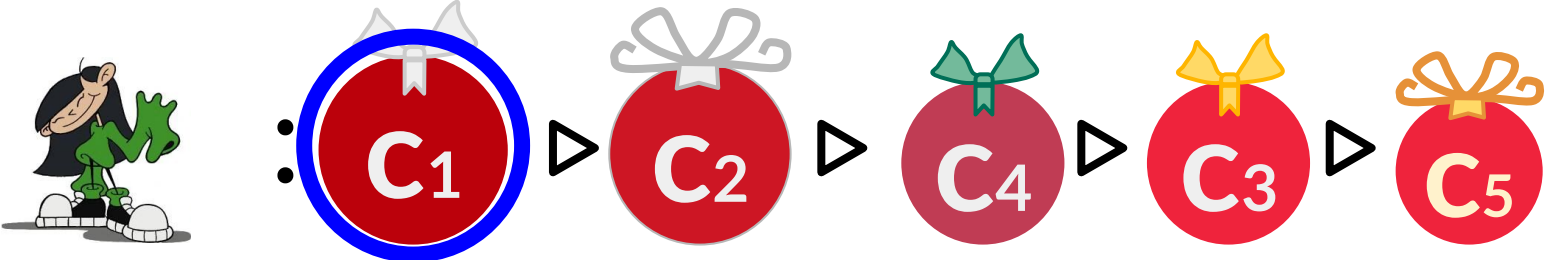
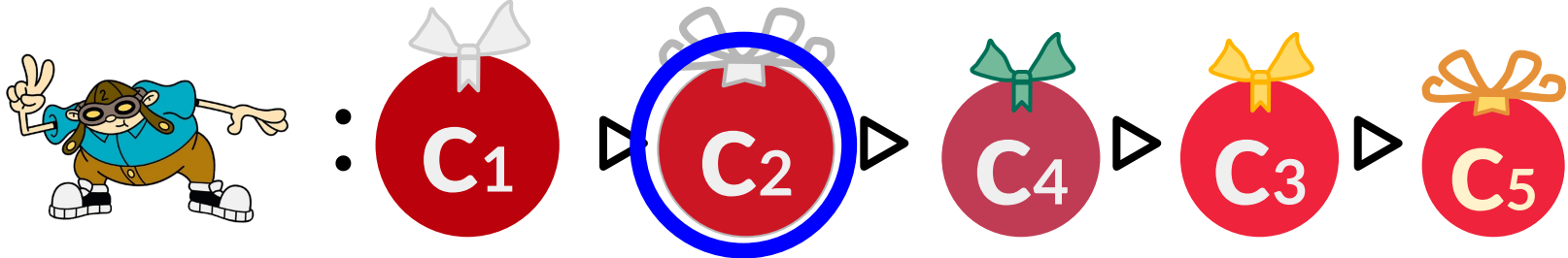
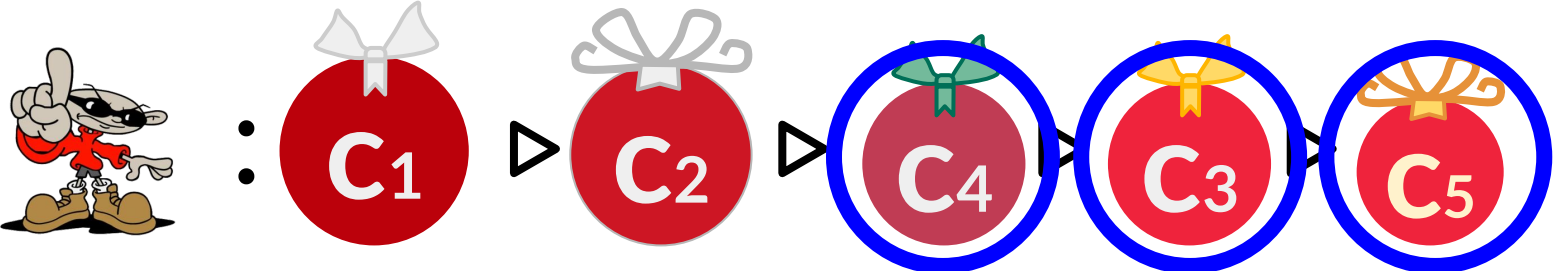
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EF1 + PO	✓ P [Hosseini et al., 2021]	?	?
MMS	✓ P [Hosseini et al., 2021]	?	?
MMS + PO	✓ P [Hosseini et al., 2021]	?	?



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<b>EFX + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
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<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	?	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	?	?
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<b>EF1</b>	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?

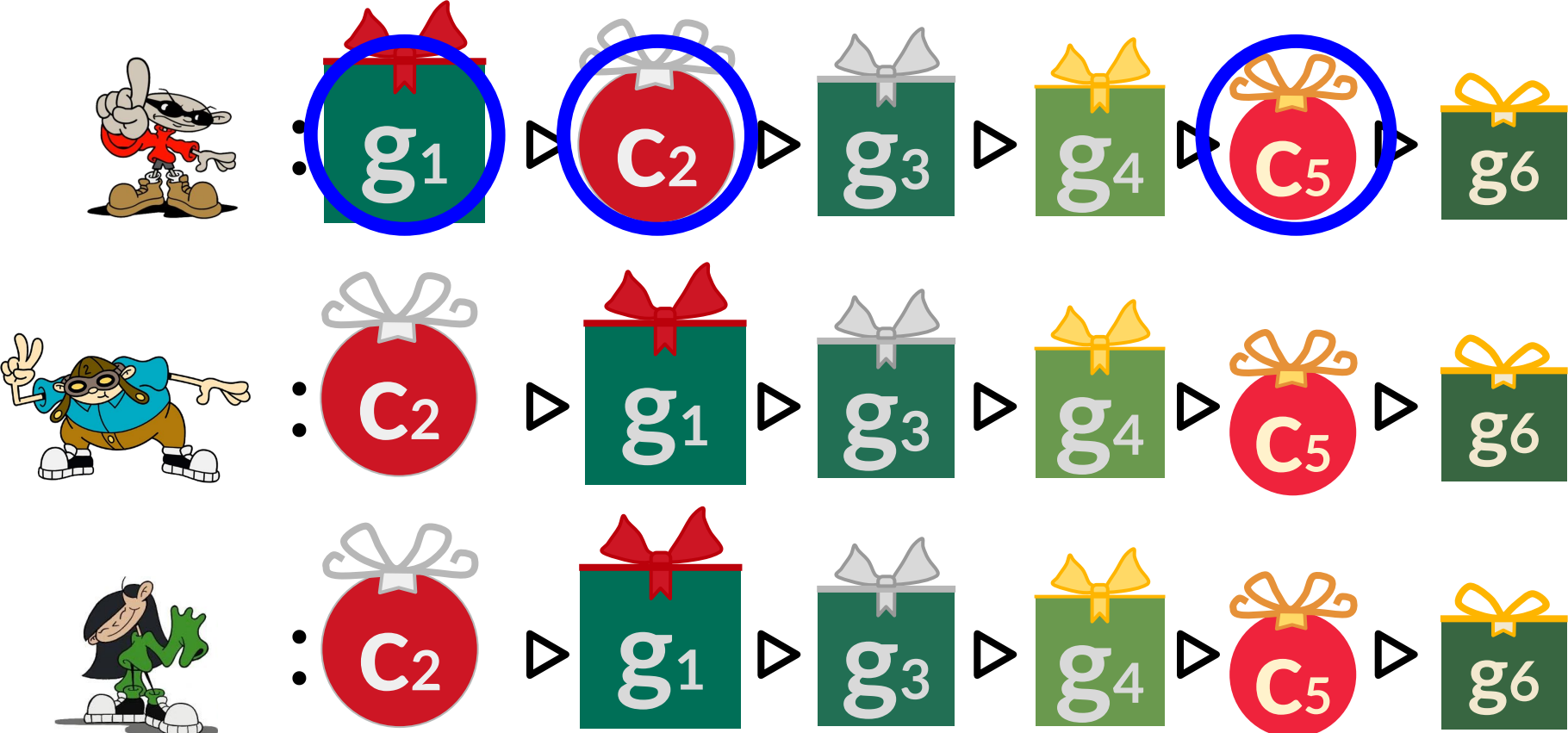
# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
<b>EFX</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>EFX + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>EF1</b>	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?

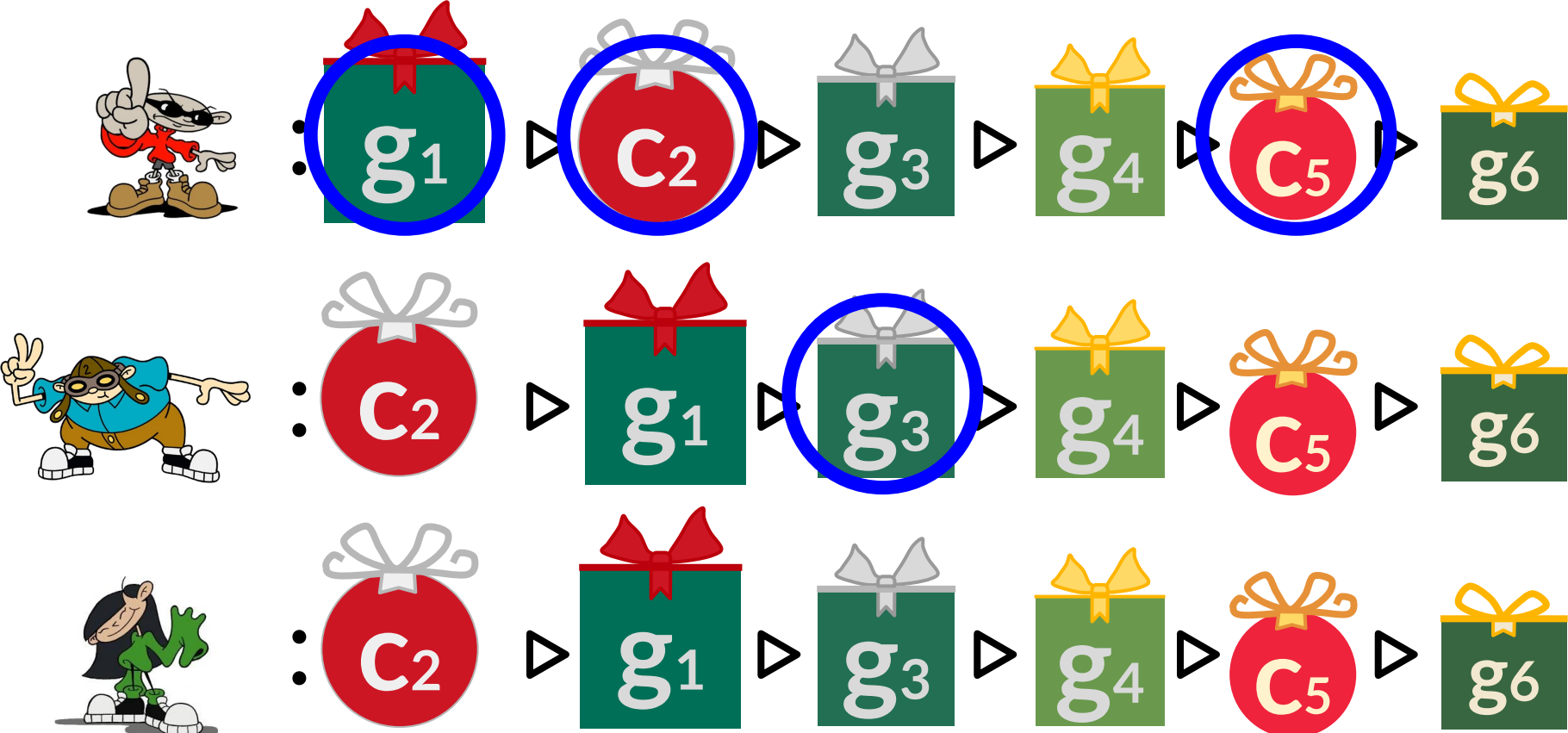
# EFX for Mixed Items



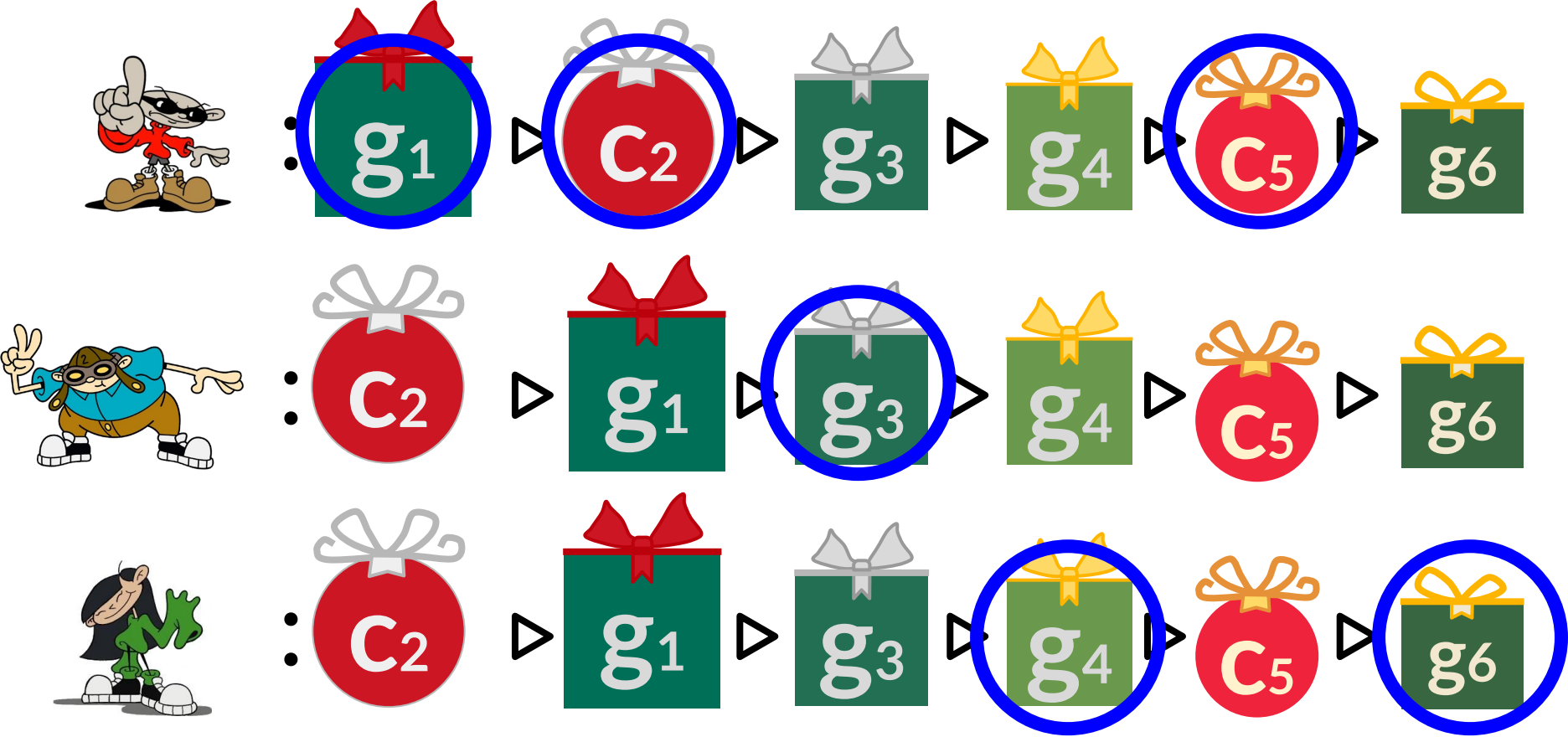
# EFX for Mixed Items



# EFX for Mixed Items



# EFX for Mixed Items

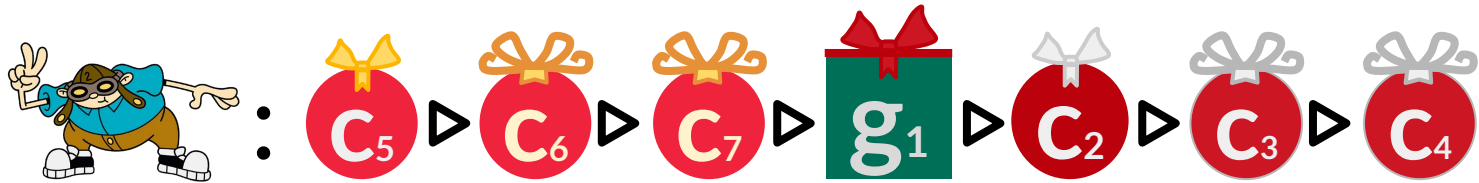


# EFX for Mixed Items (Terrible Chores)

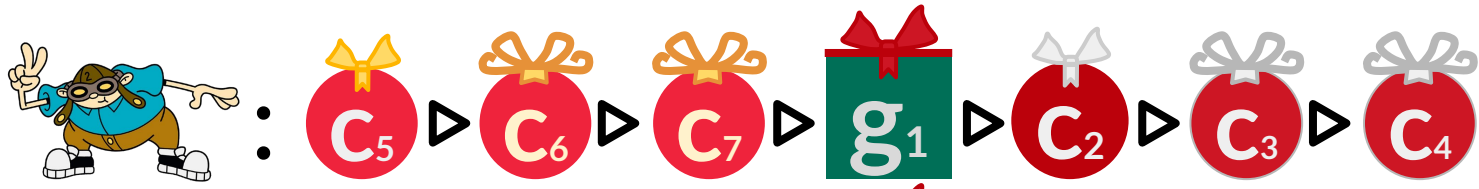
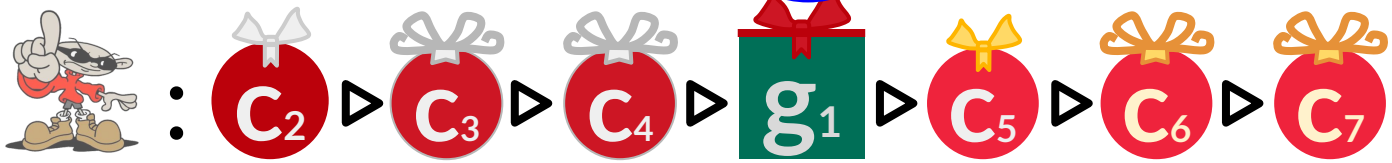
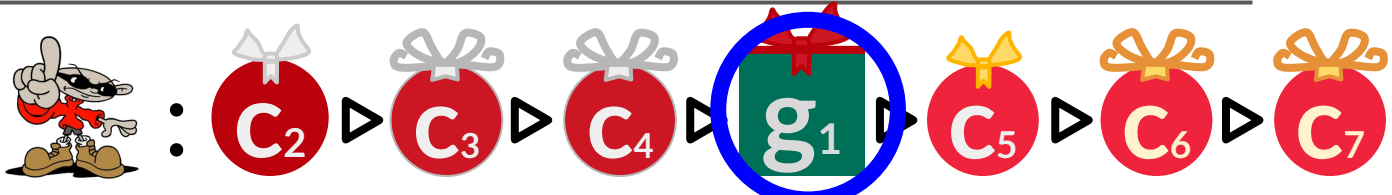
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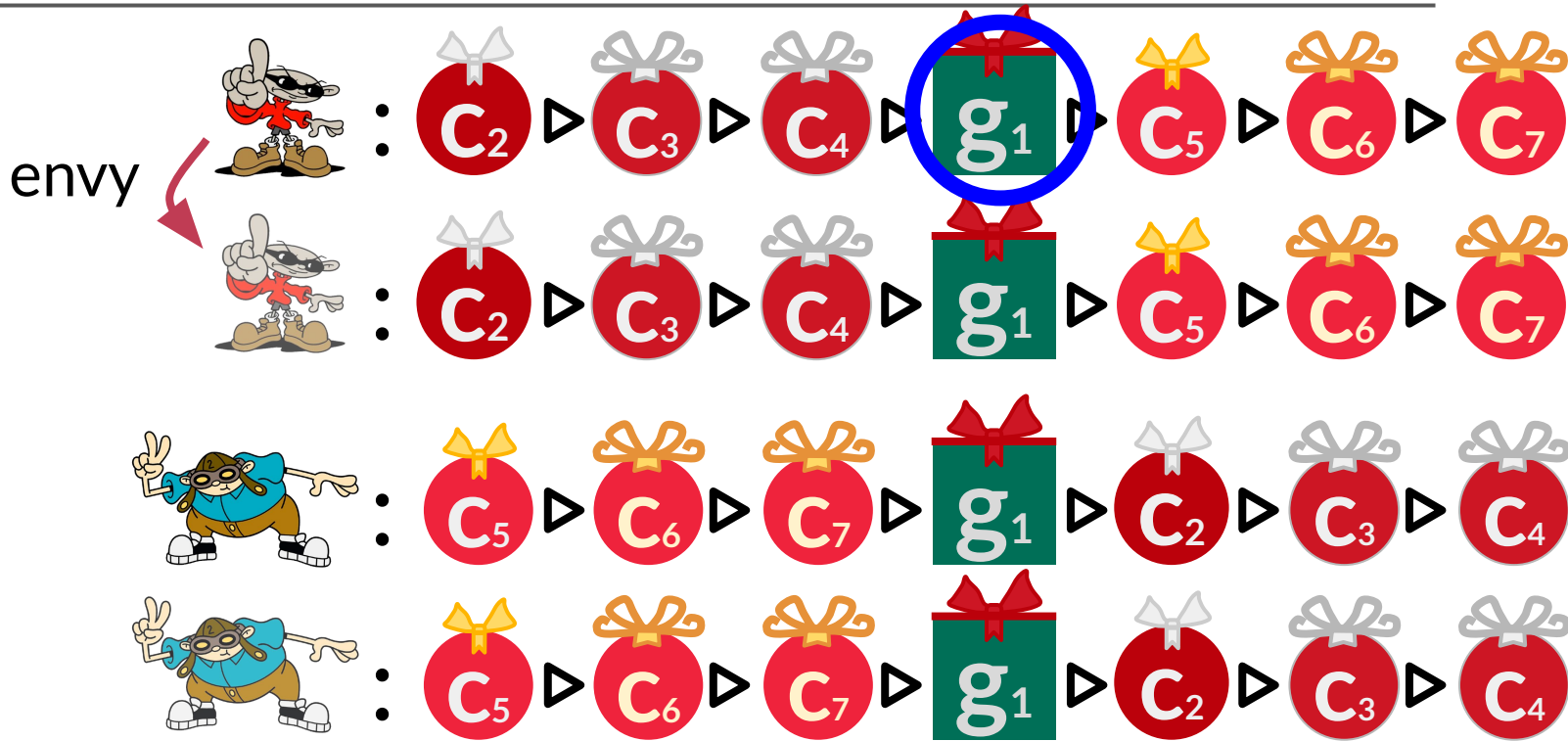
# EFX for Mixed Items (Terrible Chores)



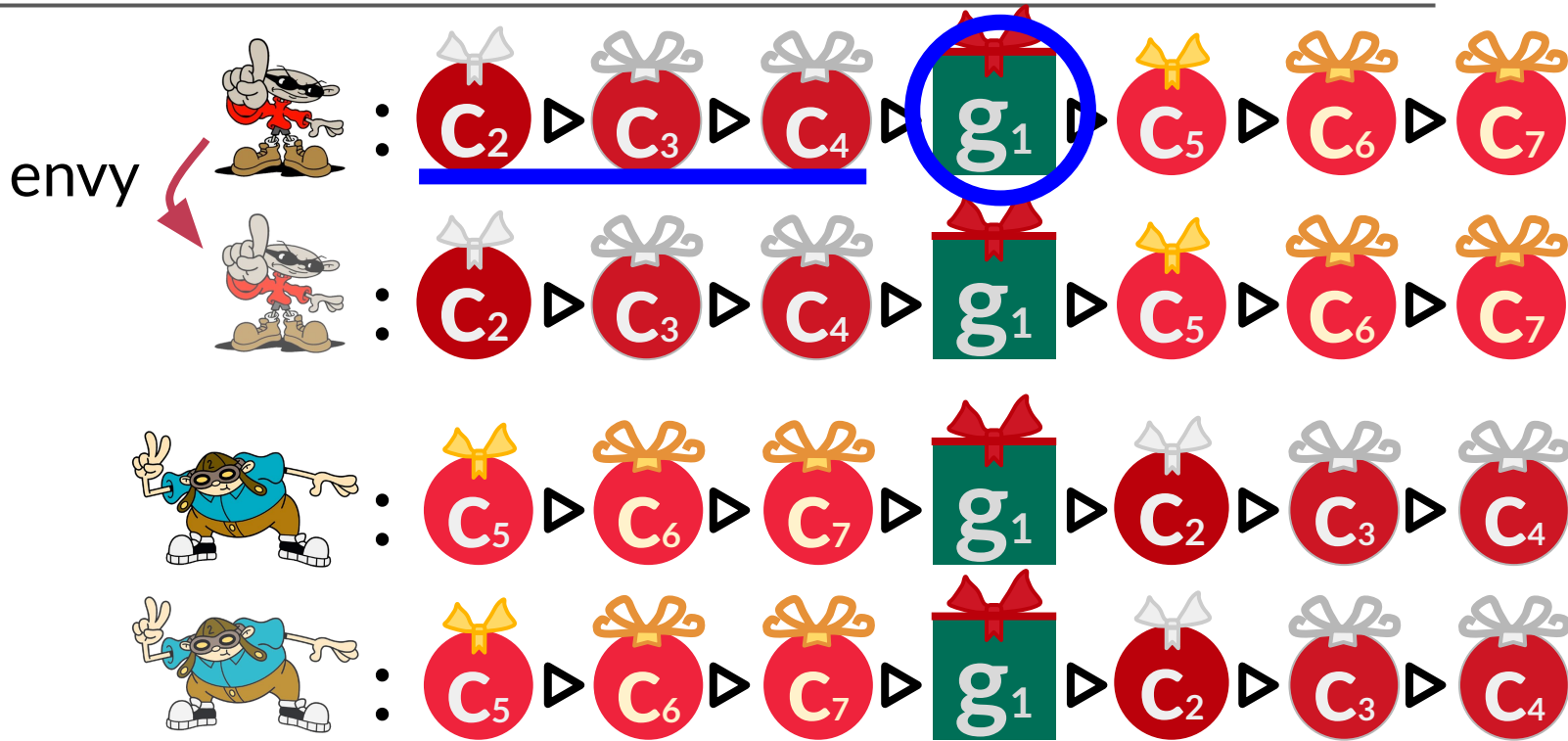
# EFX for Mixed Items (Terrible Chores)



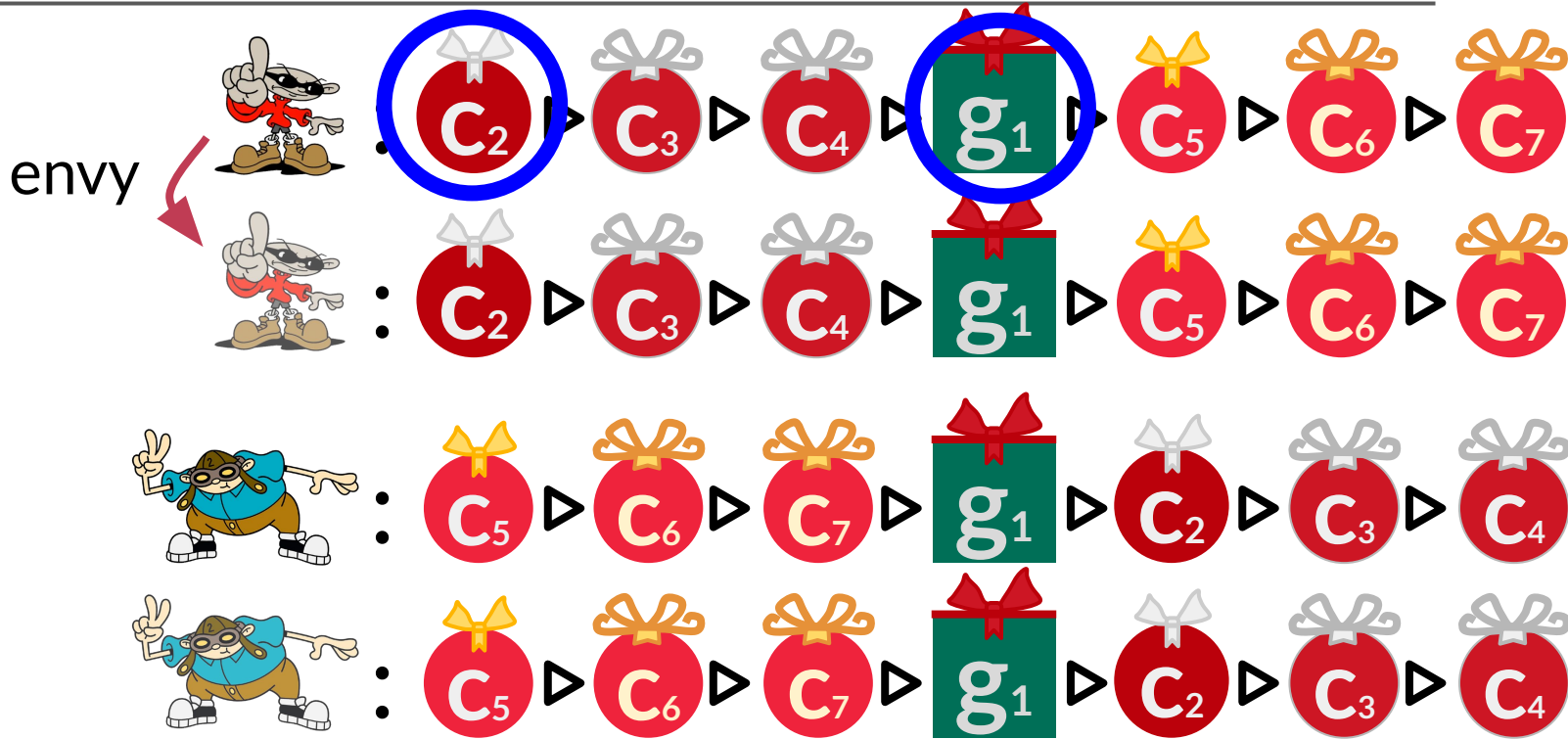
# EFX for Mixed Items (Terrible Chores)



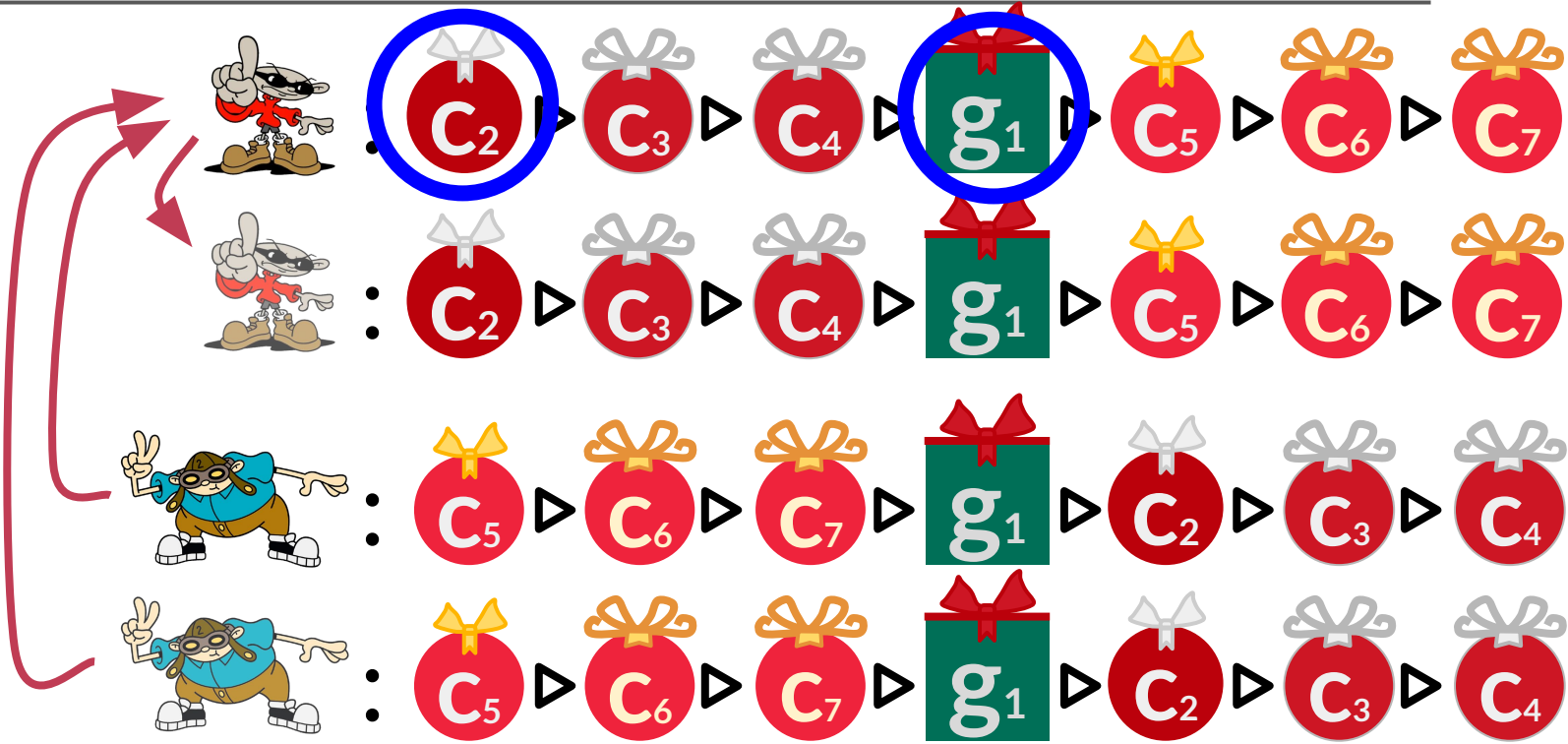
# EFX for Mixed Items (Terrible Chores)



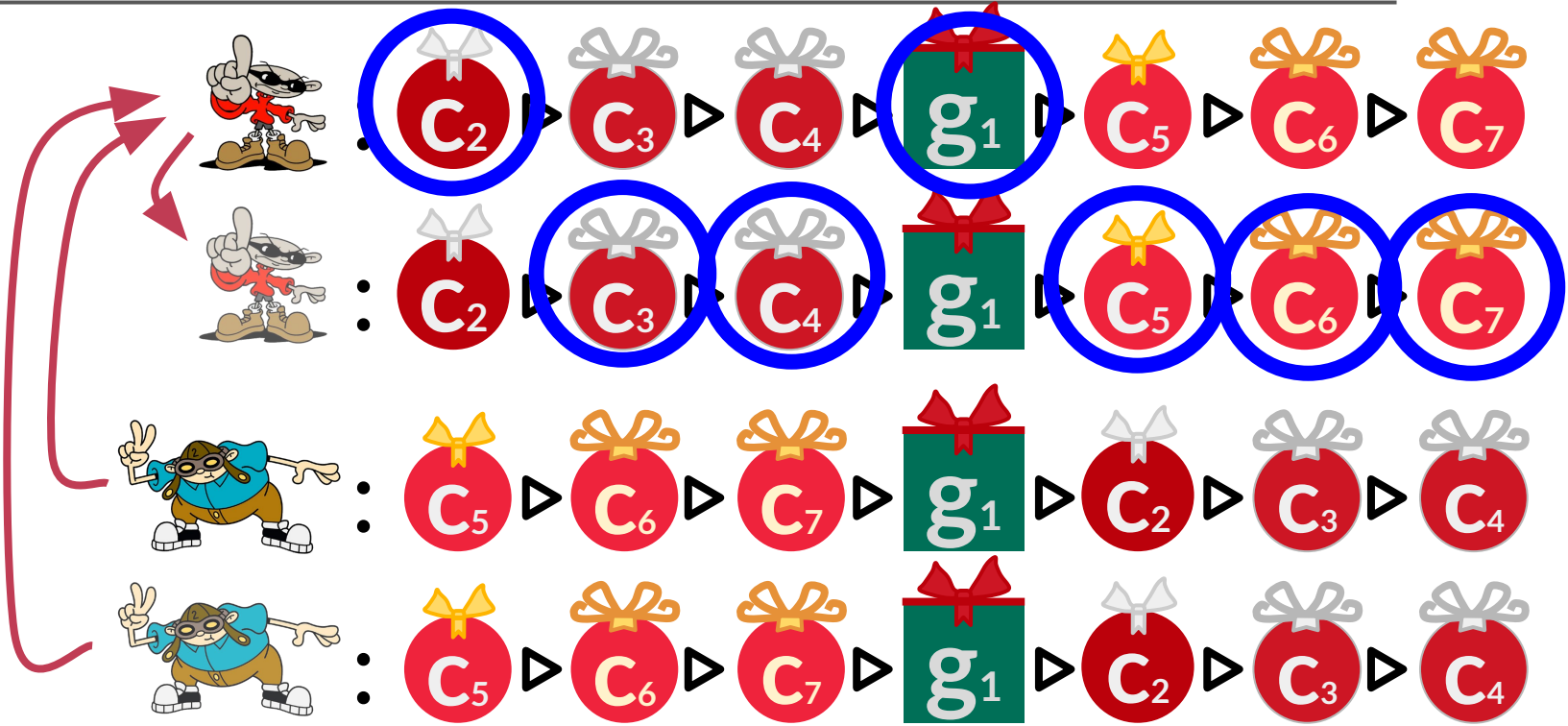
# EFX for Mixed Items (Terrible Chores)



# EFX for Mixed Items (Terrible Chores)



# EFX for Mixed Items (Terrible Chores)



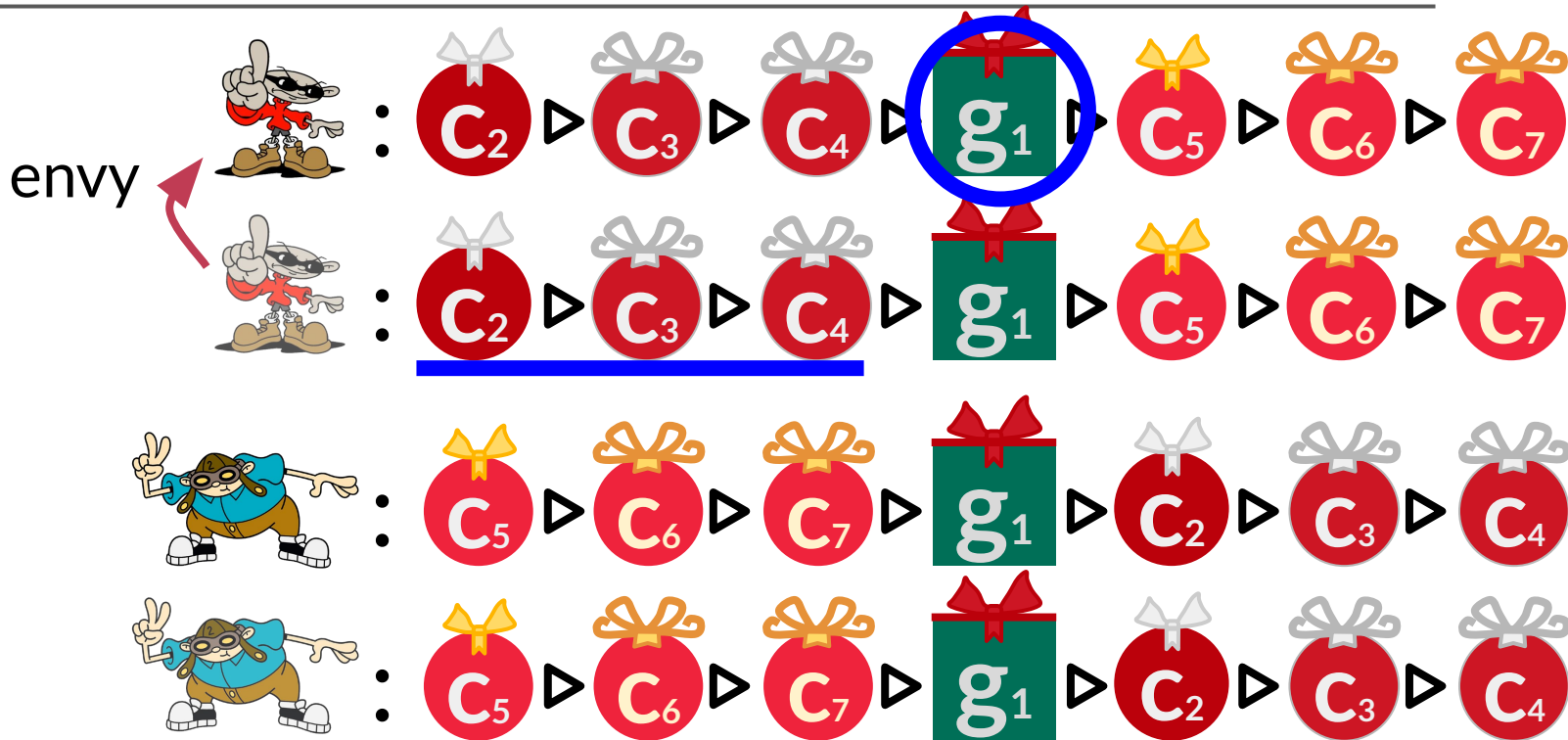
Not EFX

# EFX for Mixed Items (Terrible Chores)

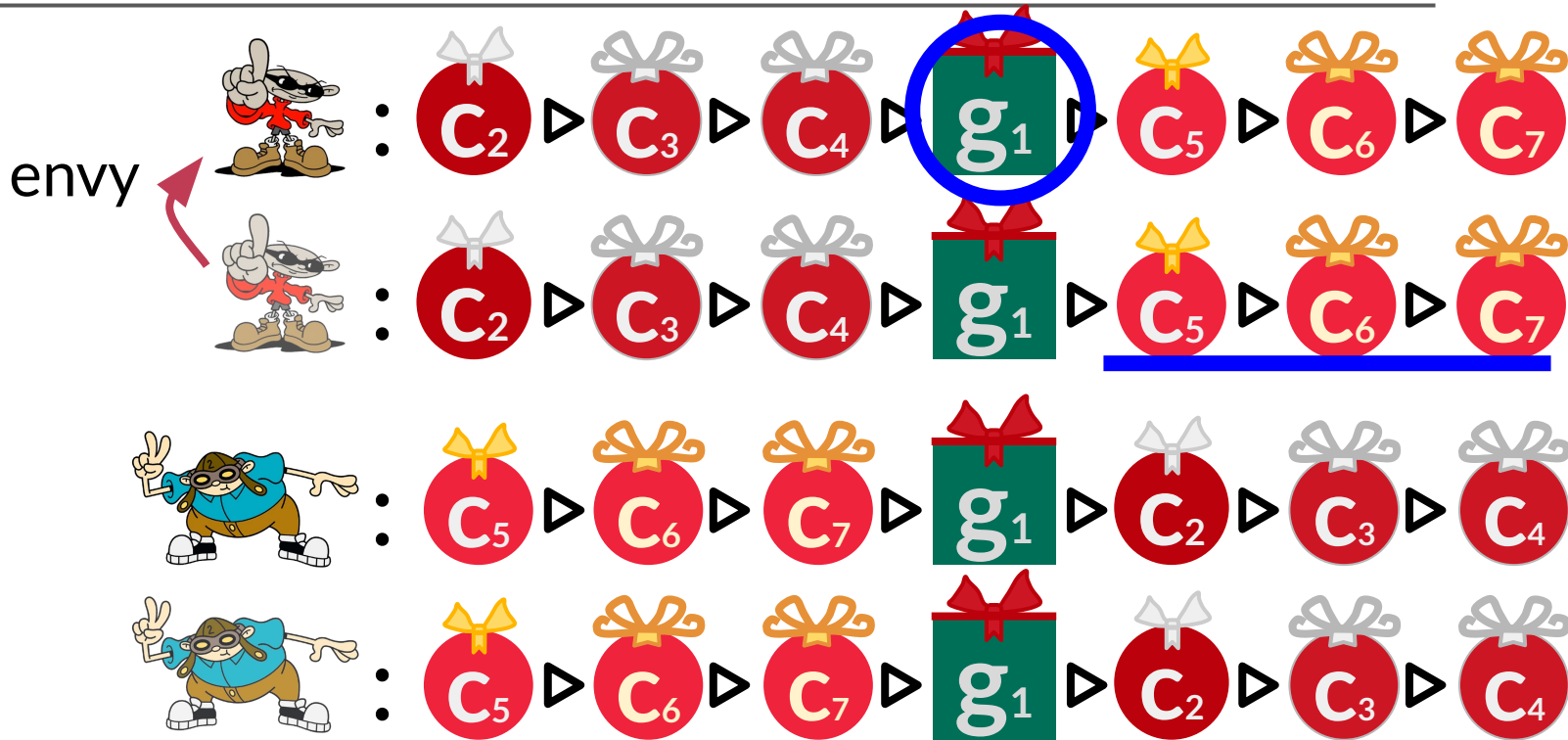




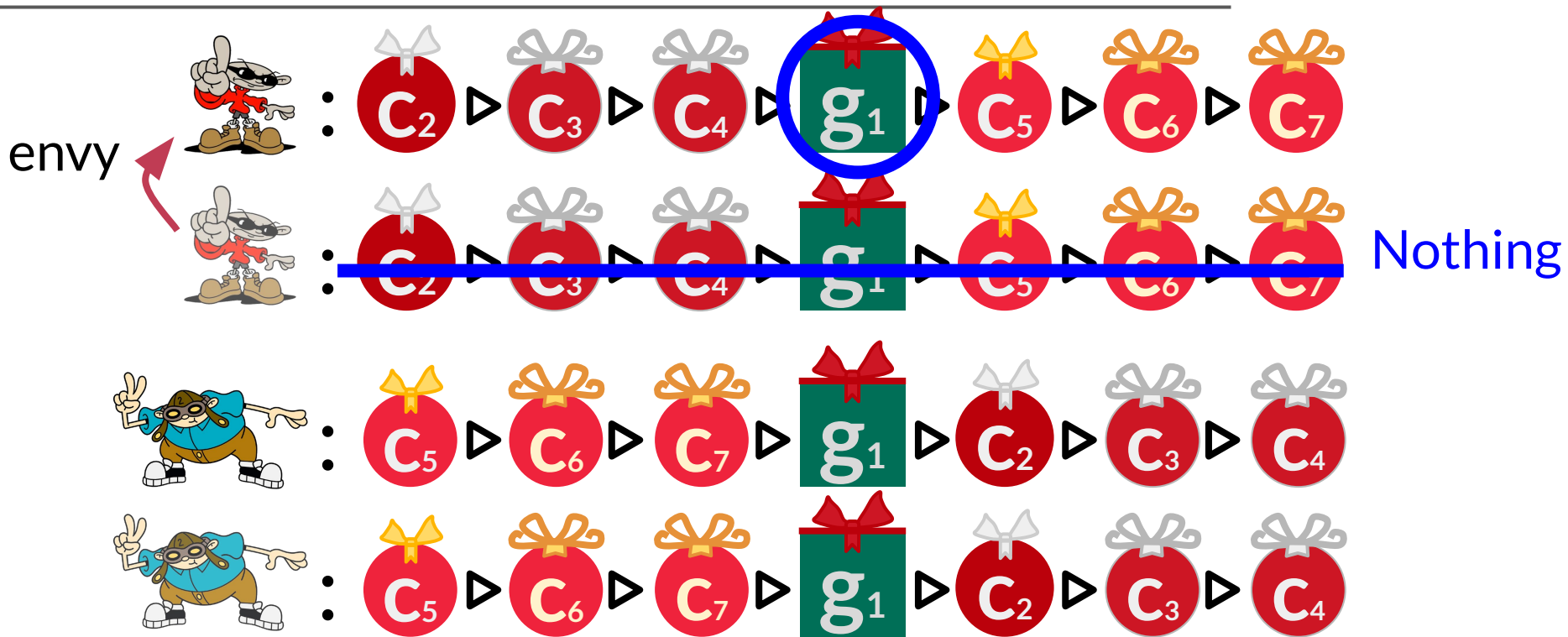
# EFX for Mixed Items (Terrible Chores)



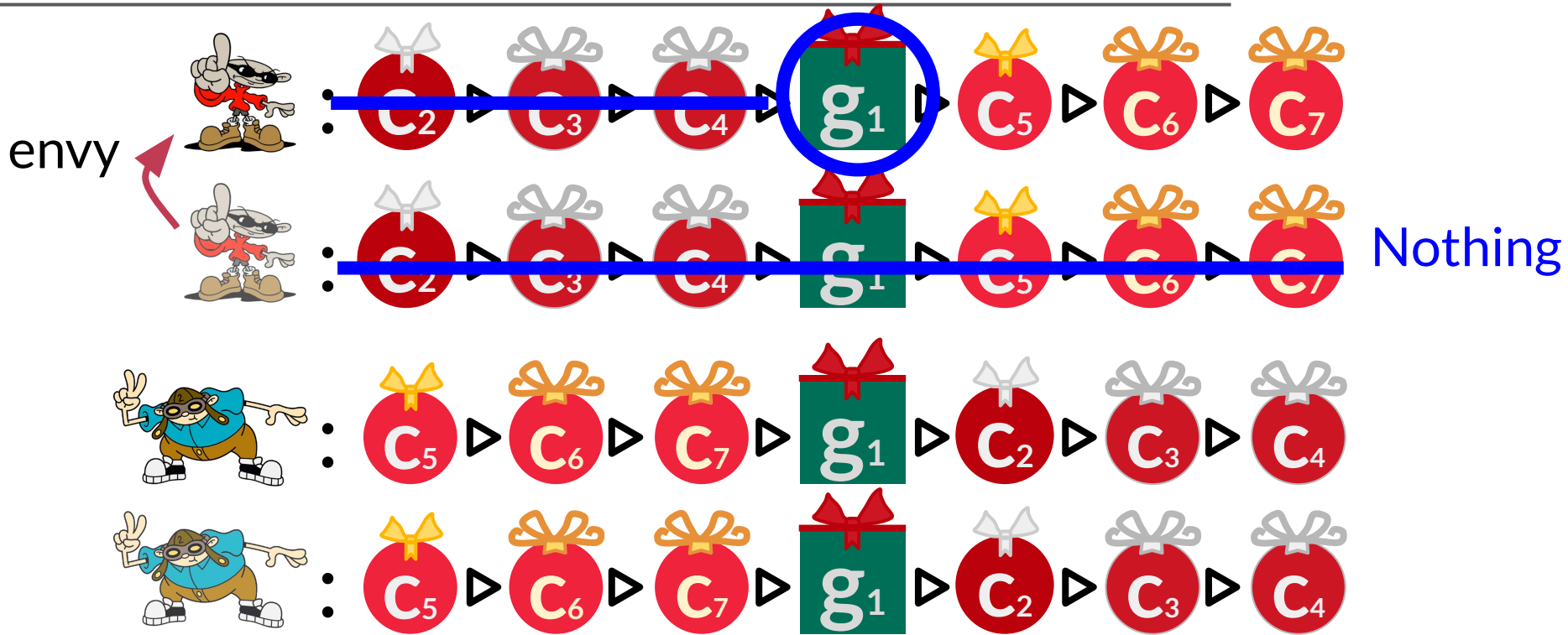
# EFX for Mixed Items (Terrible Chores)



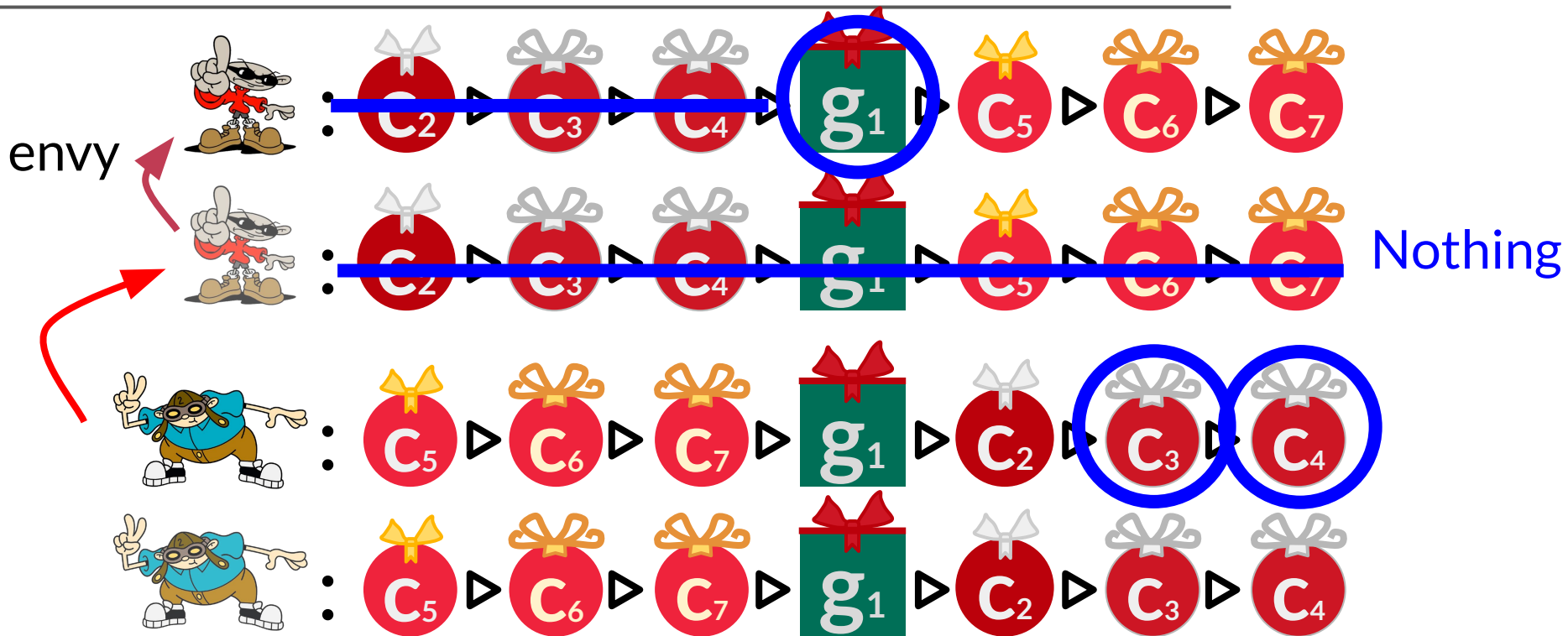
# EFX for Mixed Items (Terrible Chores)



# EFX for Mixed Items (Terrible Chores)



# EFX for Mixed Items (Terrible Chores)



Not EFX

# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
<b>EFX</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>EFX + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>EF1</b>	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?

# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
<b>EFX</b>	  [Hosseini et al., 2021]	  [Hosseini et al., 2022]	  [Hosseini et al., 2023]
<b>EFX + PO</b>	  [Hosseini et al., 2021]	  [Hosseini et al., 2022]	  [Hosseini et al., 2023]
<b>EF1</b>	  [Lipton et al., 2004]	  [Aziz et al., 2022]	  [Aziz et al., 2022]
<b>EF1 + PO</b>	  [Hosseini et al., 2021]	  [Hosseini et al., 2022]	
<b>MMS</b>	  [Hosseini et al., 2021]	  [Hosseini et al., 2022]	
<b>MMS + PO</b>	  [Hosseini et al., 2021]	  [Hosseini et al., 2022]	

# NP-completeness of EFX

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EXACT COVER BY 3-SETS (X3S).

Given  $U = \{u_1, \dots, u_m\}$  and its 3-element subsets  $S = \{S_1, \dots, S_n\}$   
is there  $K \subseteq S$  such that  $|K| = k = m/3$  and  $\bigcup_{S_j \in K} S_j = U$



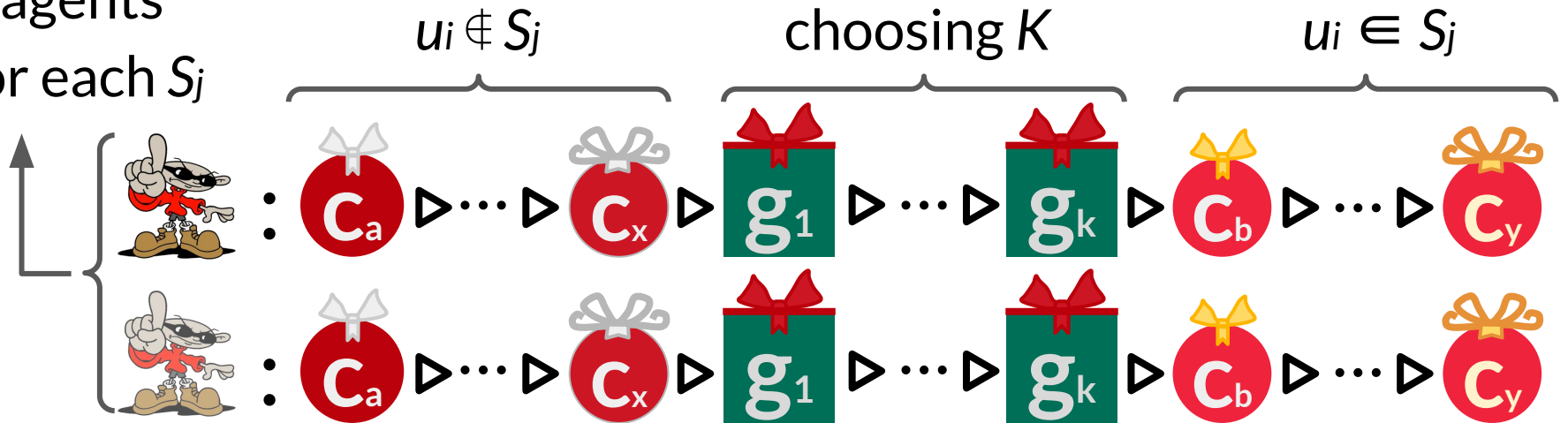
# NP-completeness of EFX

EXACT COVER BY 3-SETS (X3S).

Given  $U = \{u_1, \dots, u_m\}$  and its 3-element subsets  $S = \{S_1, \dots, S_n\}$   
is there  $K \subseteq S$  such that  $|K| = k = m/3$  and  $\cup_{S_j \in K} S_j = U$

2 agents

for each  $S_j$






# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
<b>EFX</b>	 <b>P</b> [Hosseini et al., 2021]	 <b>P</b> [Hosseini et al., 2022]	 <b>deciding?</b> [Hosseini et al., 2023]
<b>EFX + PO</b>	 <b>P</b> [Hosseini et al., 2021]	 <b>P</b> [Hosseini et al., 2022]	 <b>deciding?</b> [Hosseini et al., 2023]
<b>EF1</b>	 <b>P</b> [Lipton et al., 2004]	 <b>P</b> [Aziz et al., 2022]	 <b>P</b> [Aziz et al., 2022]
<b>EF1 + PO</b>	 <b>P</b> [Hosseini et al., 2021]	 <b>P</b> [Hosseini et al., 2022]	
<b>MMS</b>	 <b>P</b> [Hosseini et al., 2021]	 <b>P</b> [Hosseini et al., 2022]	
<b>MMS + PO</b>	 <b>P</b> [Hosseini et al., 2021]	 <b>P</b> [Hosseini et al., 2022]	

# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
<b>EFX</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EFX + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EF1</b>	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?

# Additive Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
EFX	?	?	?
EFX + PO	?	?	?
EF1	 <p>P</p> <p>[Lipton et al., 2004]</p>	 <p>P</p> <p>[Aziz et al., 2022]</p>	 <p>P</p> <p>[Aziz et al., 2022]</p>
EF1 + PO	 <p>pseudo-P</p> <p>[Caragiannis et al., 2019]</p>	?	?
MMS	 <p>deciding?</p> <p>[Procaccia &amp; Wang, 2014]</p>	 <p>deciding?</p> <p>[Aziz et al., 2017]</p>	 <p>deciding?</p> <p>[Procaccia &amp; Wang, 2014]</p>
MMS + PO	 <p>deciding?</p> <p>[Procaccia &amp; Wang, 2014]</p>	 <p>deciding?</p> <p>[Aziz et al., 2017]</p>	 <p>deciding?</p> <p>[Procaccia &amp; Wang, 2014]</p>

# Additive Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
EFX	?	?	✗ NP-c. [Hosseini et al., 2023]
EFX + PO	?	?	✗ NP-h. [Hosseini et al., 2023]
EF1	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
EF1 + PO	✓ pseudo-P [Caragiannis et al., 2019]	?	?
MMS	✗ deciding? [Procaccia & Wang, 2014]	✗ deciding? [Aziz et al., 2017]	✗ deciding? [Procaccia & Wang, 2014]
MMS + PO	✗ deciding? [Procaccia & Wang, 2014]	✗ deciding? [Aziz et al., 2017]	✗ deciding? [Procaccia & Wang, 2014]

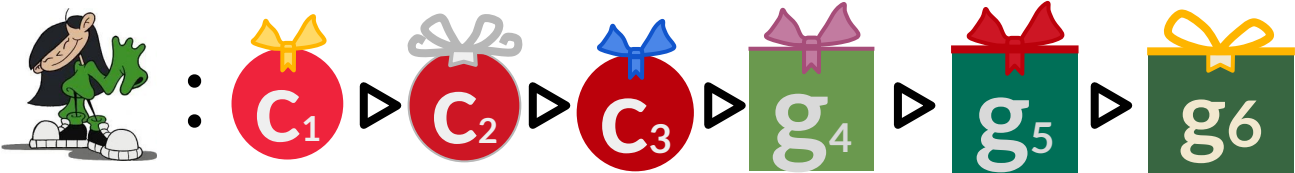
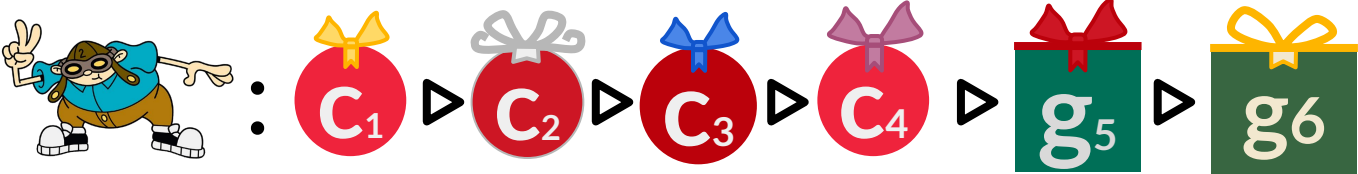
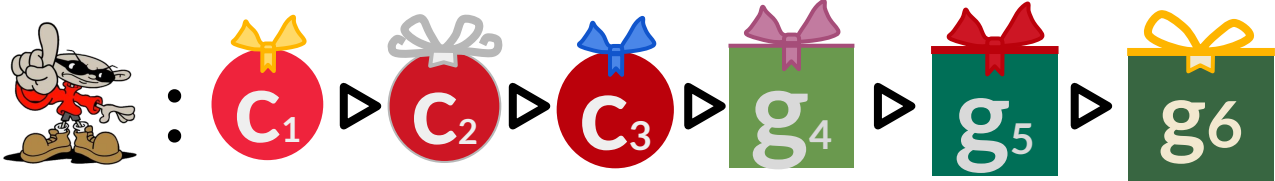
# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
<b>EFX</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EFX + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EF1</b>	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?

# Lexicographic Preferences State of Knowledge

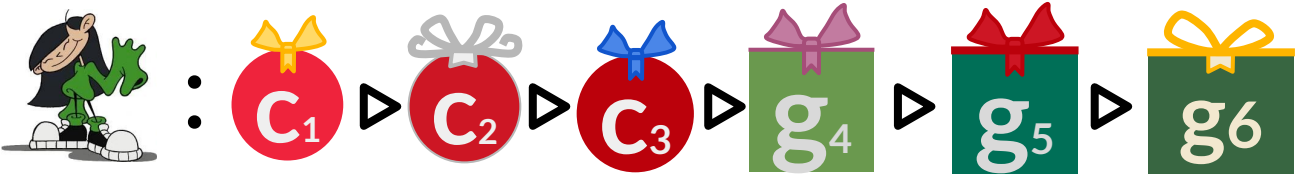
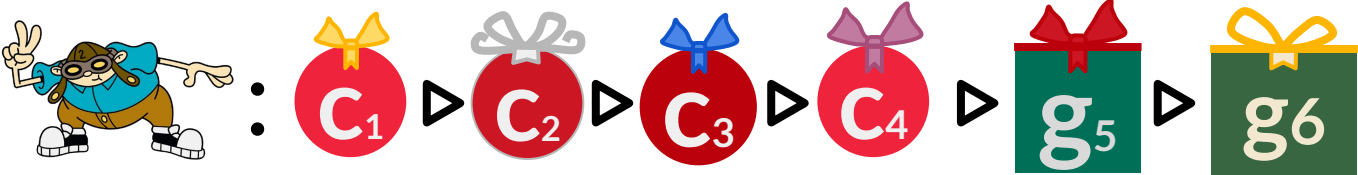
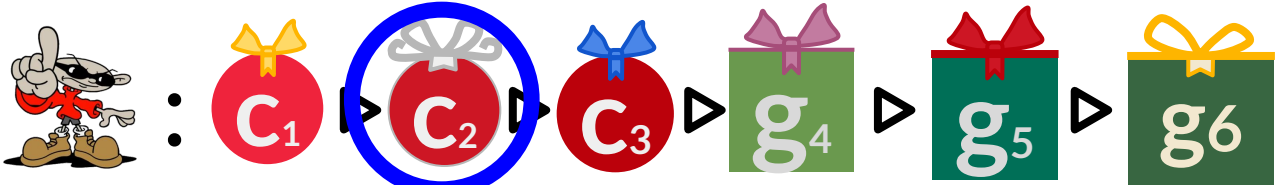
	Only Goods	Only Chores	Mixed
<b>EFX</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EFX + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EF1</b>	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?

# EFX+PO for Separable Instances

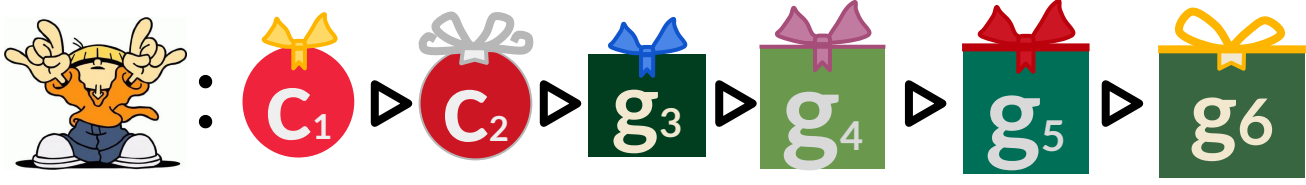
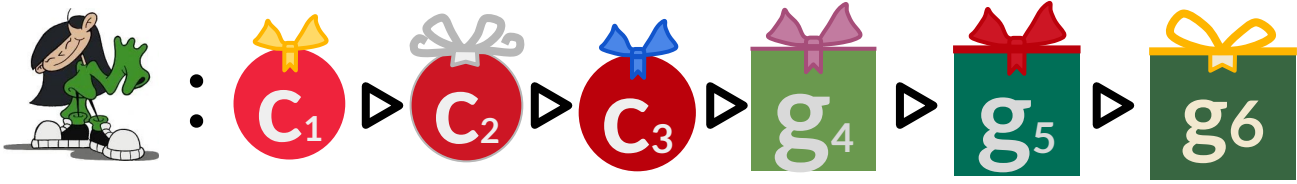
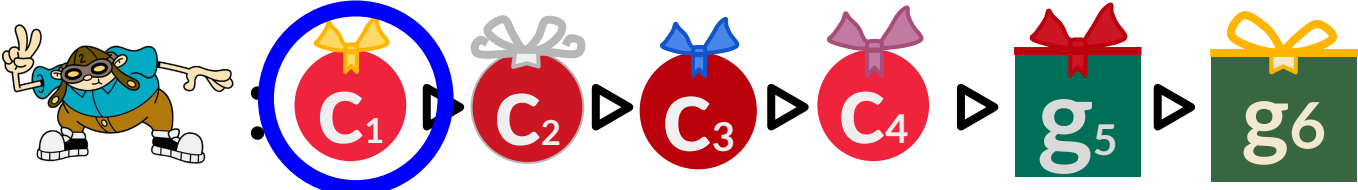
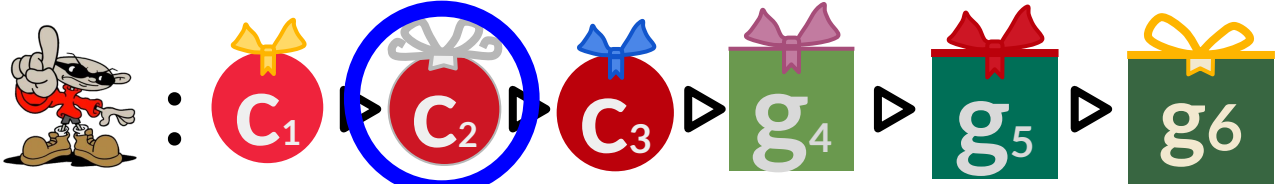




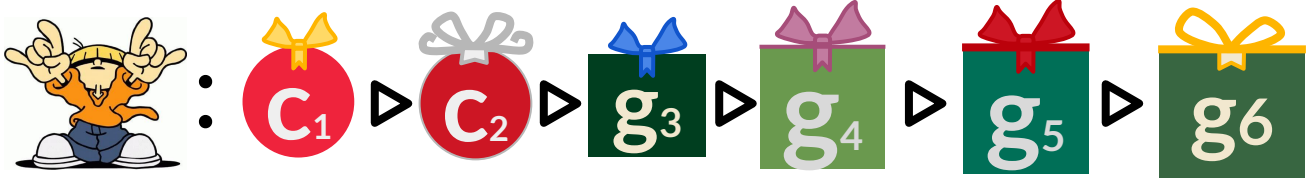
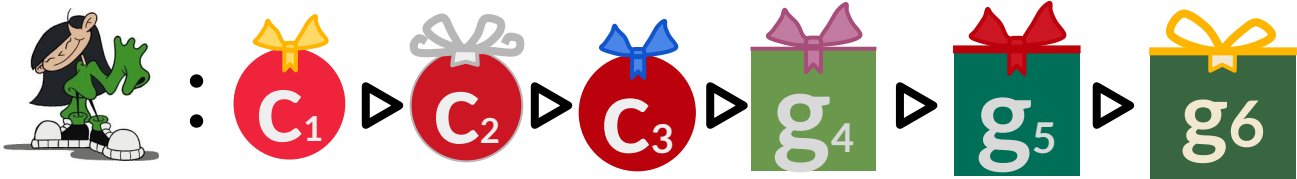
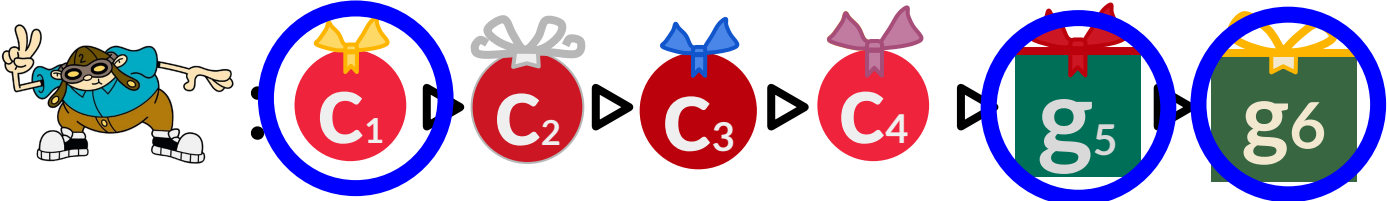
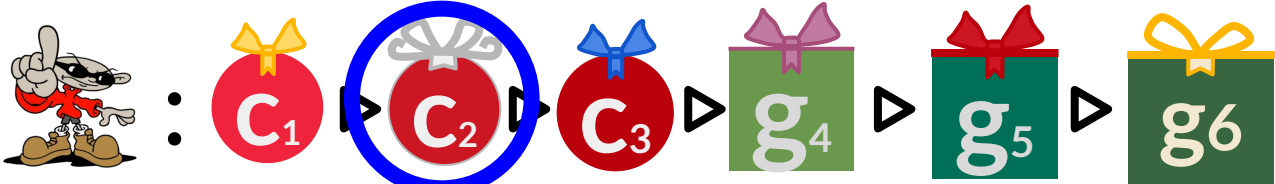
# EFX+PO for Separable Instances



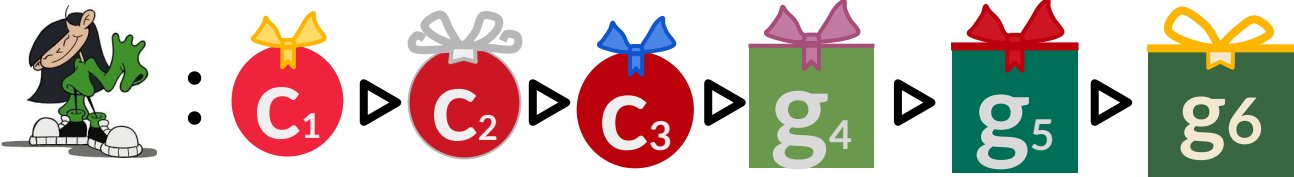
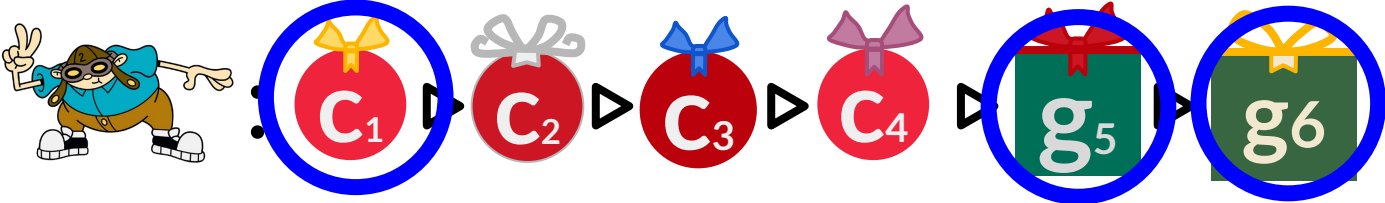
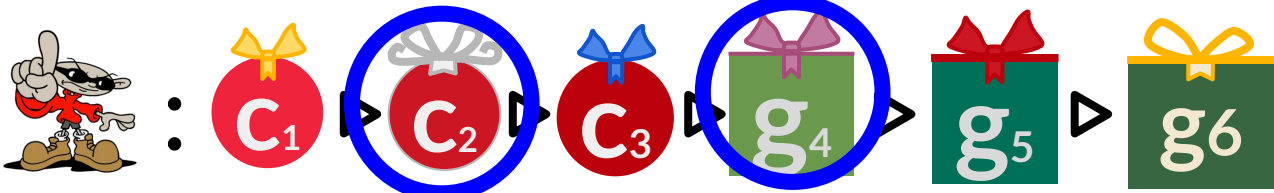
# EFX+PO for Separable Instances



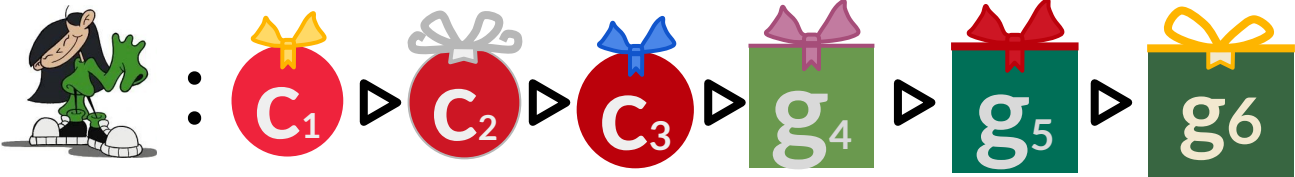
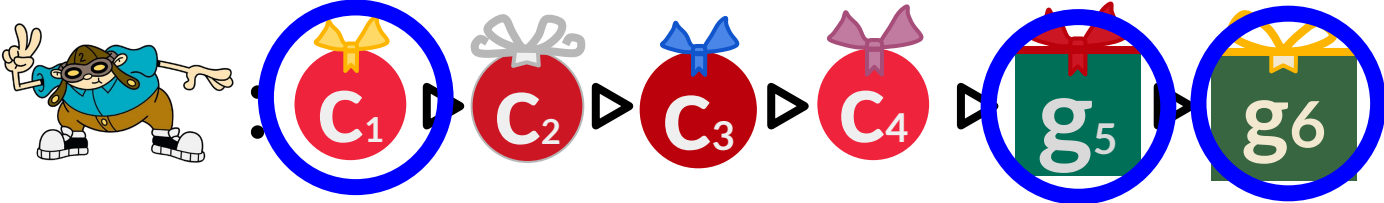
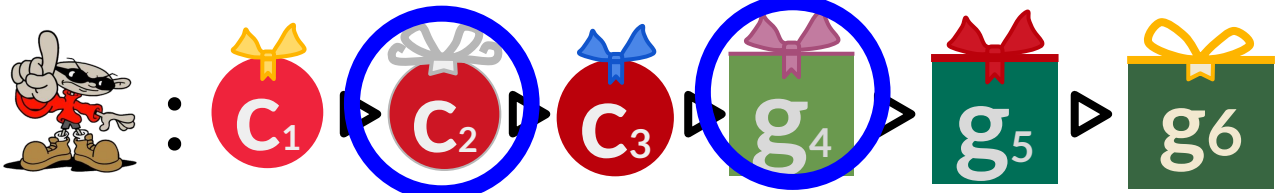
# EFX+PO for Separable Instances



# EFX+PO for Separable Instances



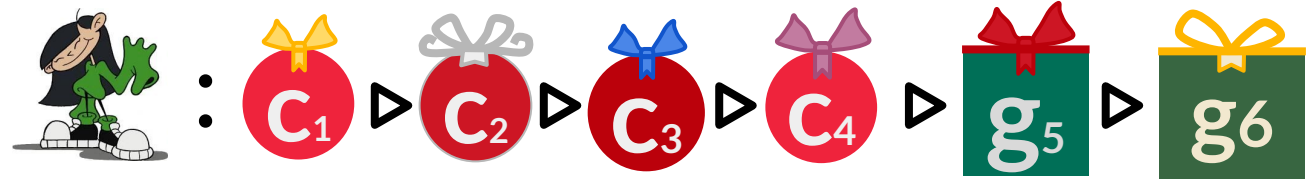
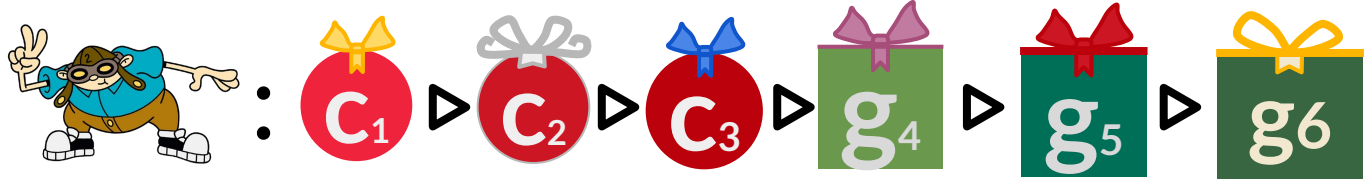
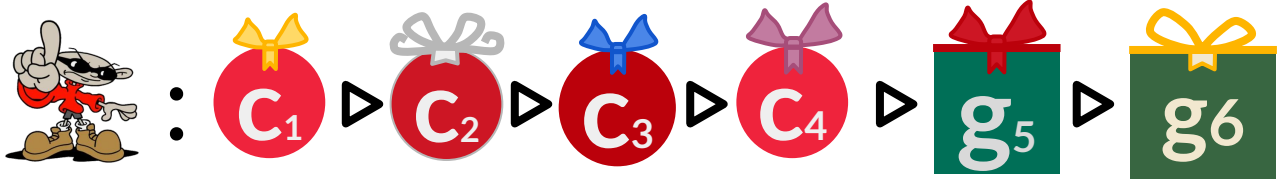
# EFX+PO for Separable Instances



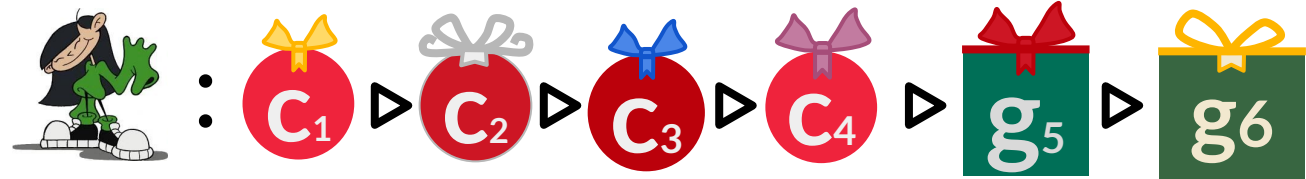
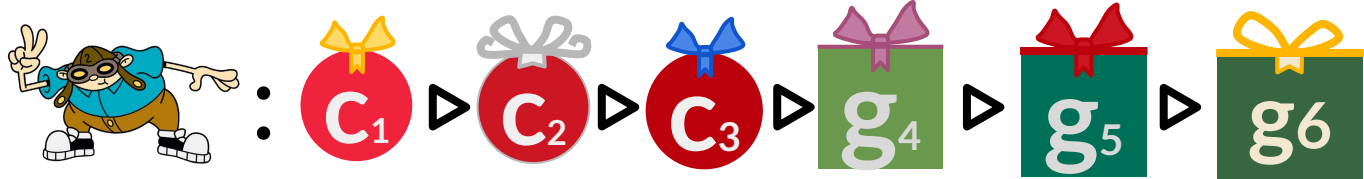
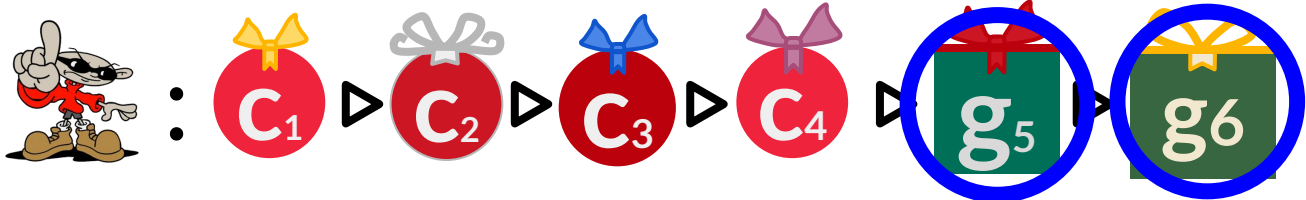
# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
<b>EFX</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EFX + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EF1</b>	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?

# MMS+PO for Lexicographic Instances

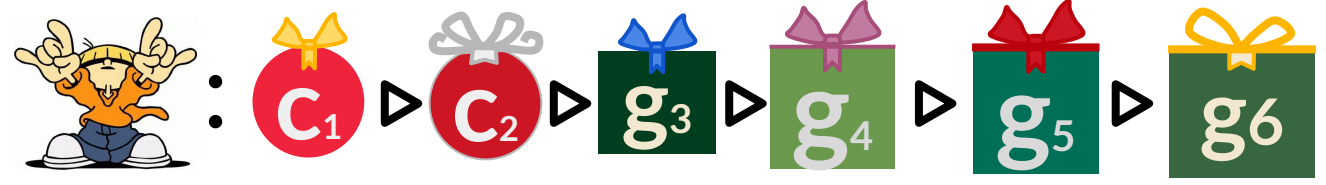
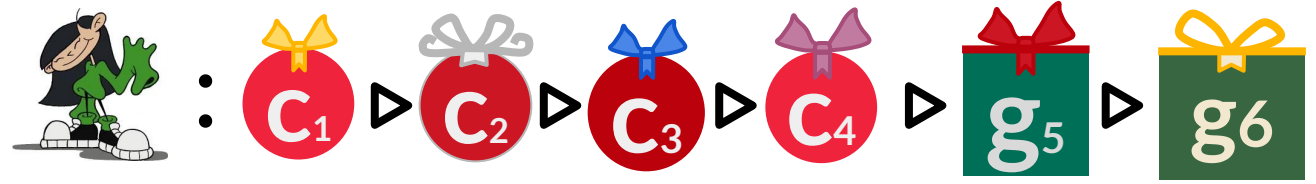
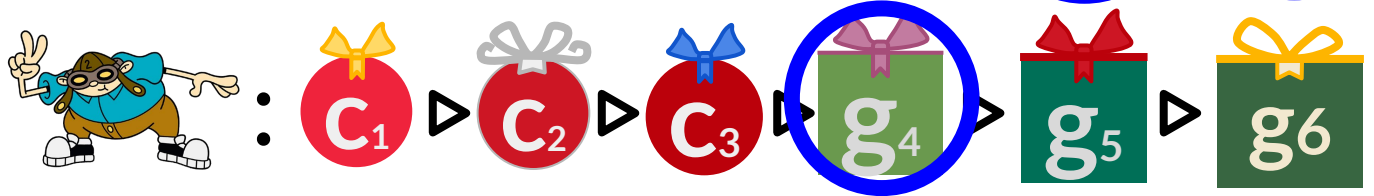
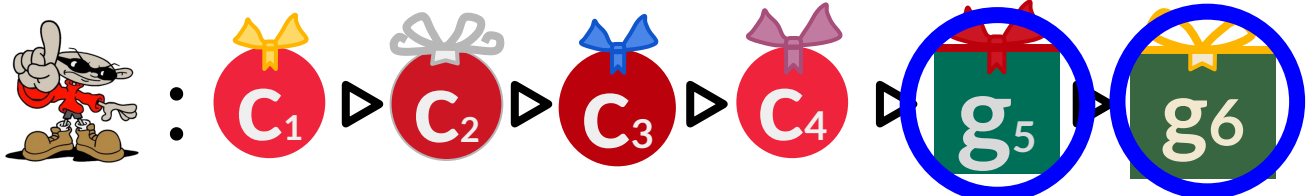


# MMS+PO for Lexicographic Instances

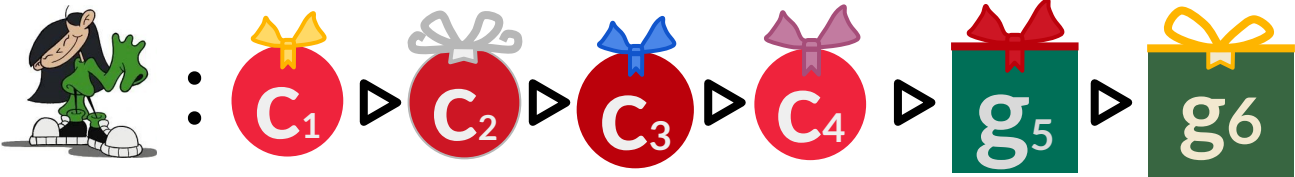
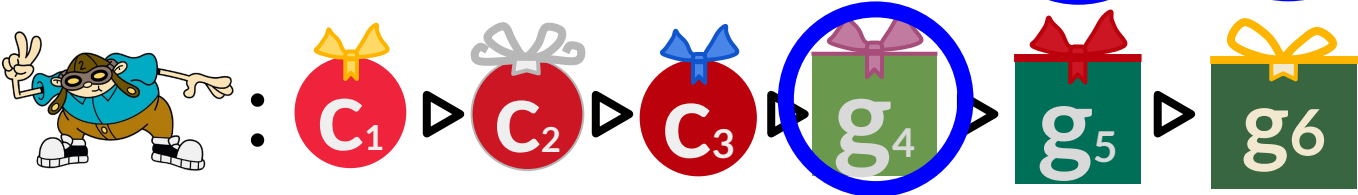
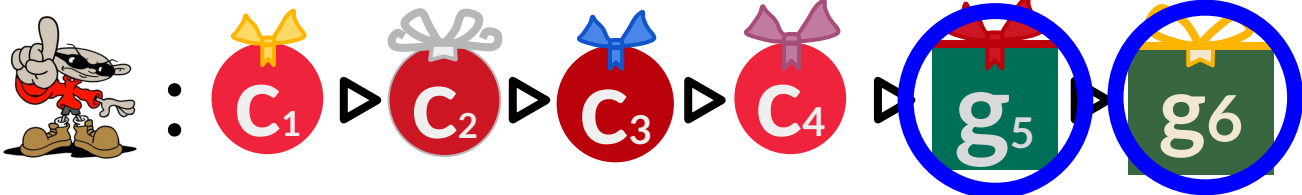




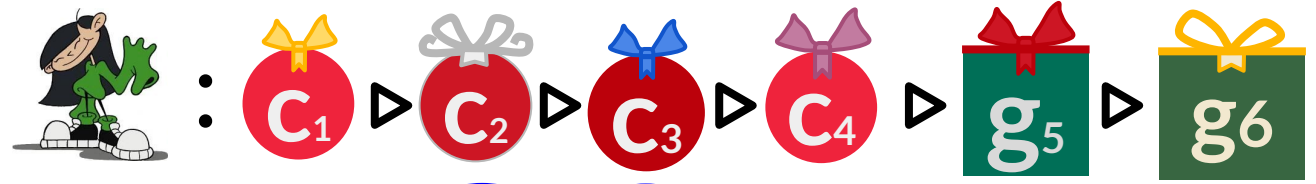
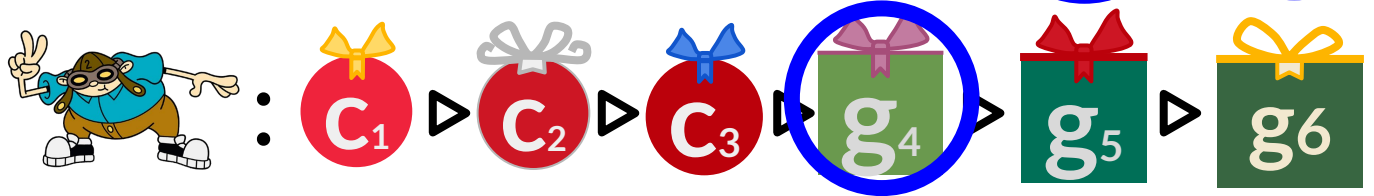
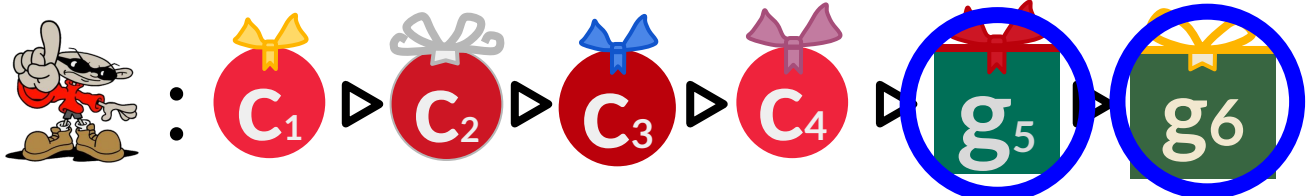
# MMS+PO for Lexicographic Instances



# MMS+PO for Lexicographic Instances



# MMS+PO for Lexicographic Instances



# MMS+PO for Lexicographic Instances



# MMS for lexicographic preferences

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$$\text{MMS}_{\text{char}} = \{ \text{gift}_1, \text{gift}_1, \text{gift}_2, \dots, \text{gift}_m \}$$

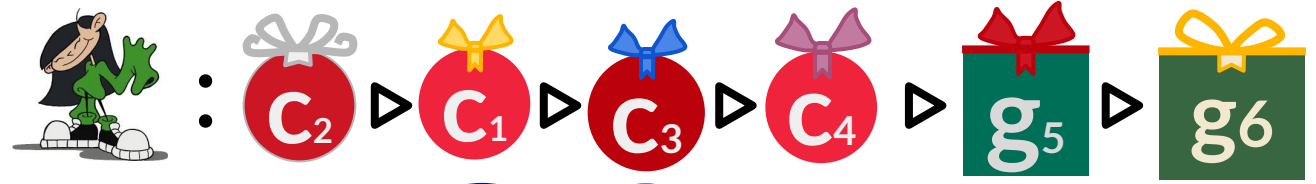
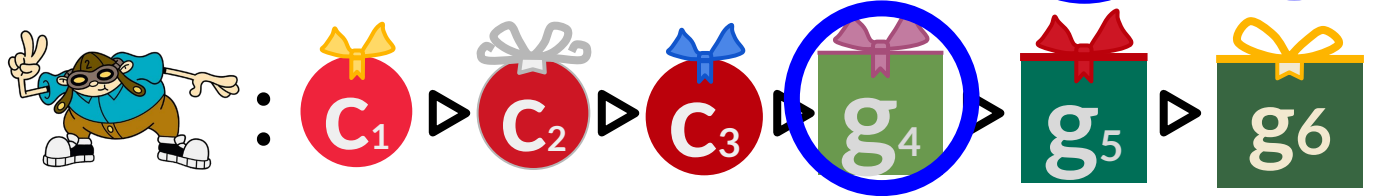
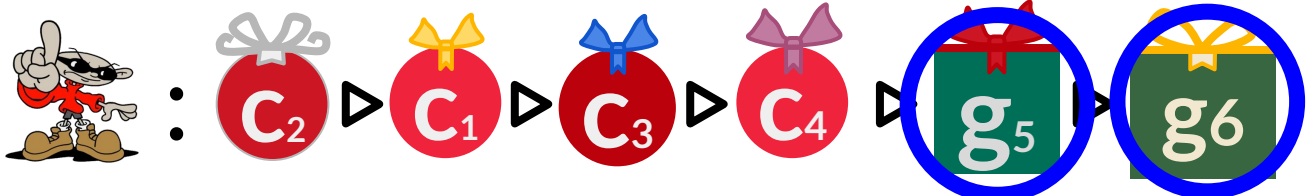


$$\text{MMS}_{\text{char}} = \{ \text{gift}_n, \text{gift}_{n+1}, \dots, \text{gift}_m \}$$

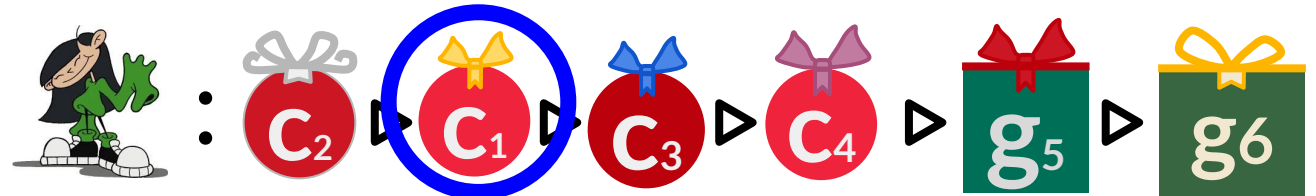
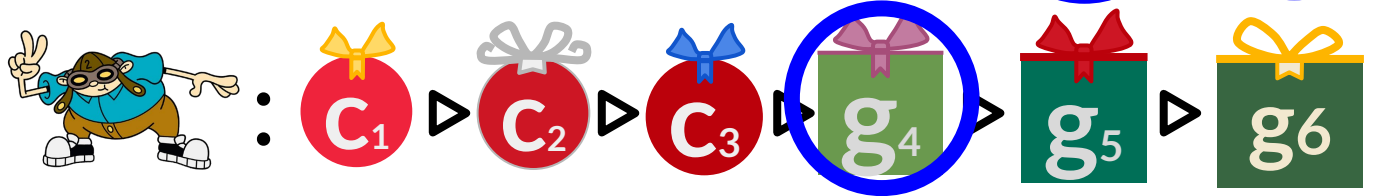
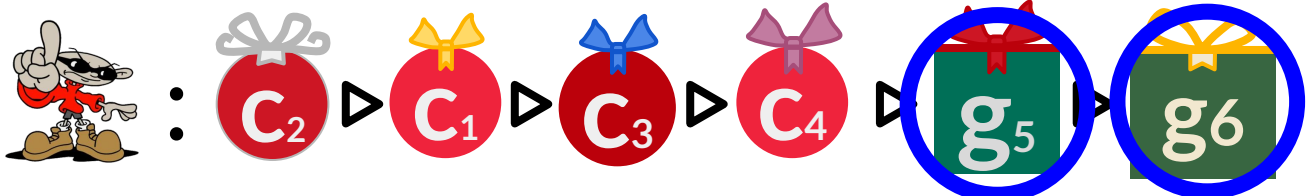
# MMS+PO for Lexicographic Instances



# MMS+PO for Lexicographic Instances



# MMS+PO for Lexicographic Instances





# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
<b>EFX</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EFX + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
<b>EF1</b>	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?

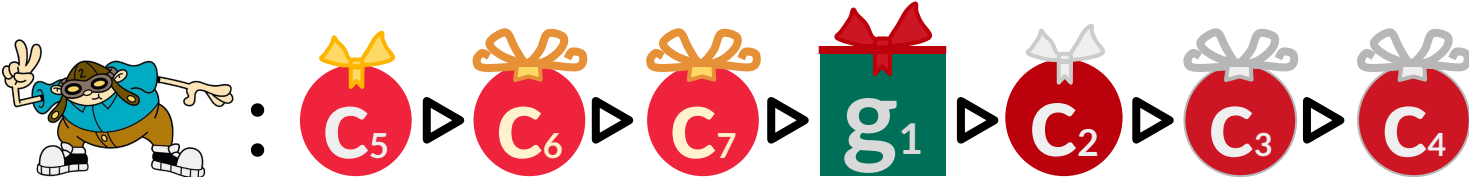
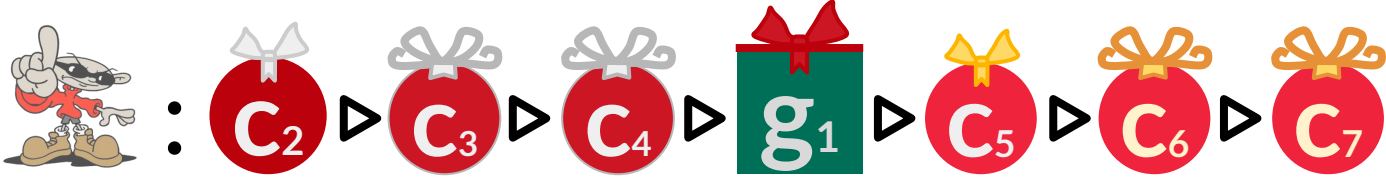
# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
<b>EFX</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✗ NP-c. [Hosseini et al., 2023]
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<b>EF1</b>	✓ P [Lipton et al., 2004]	✓ P [Aziz et al., 2022]	✓ P [Aziz et al., 2022]
<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✓ P [Hosseini et al., 2023]
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✓ P [Hosseini et al., 2023]

# Lexicographic Preferences State of Knowledge

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<b>EF1 + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	?
<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✓ P [Hosseini et al., 2023]
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✓ P [Hosseini et al., 2023]

# EF1+PO for Mixed Items - an open problem



# EF1+PO for Mixed Items - an open problem



# EF1+PO for Mixed Items - an open problem

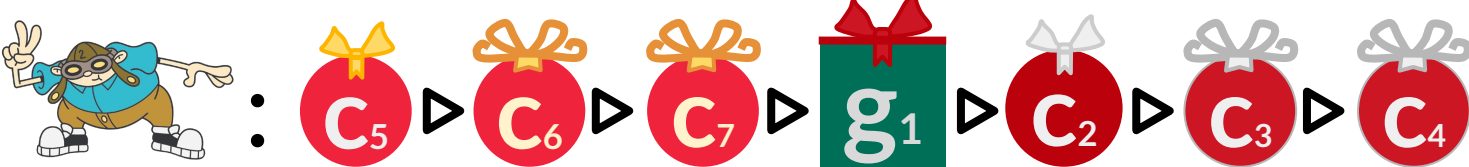
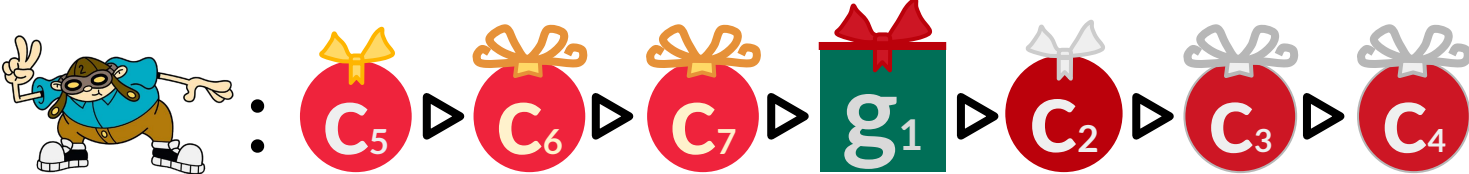


# EF1+PO for Mixed Items - an open problem



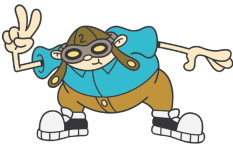
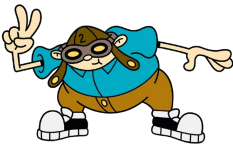
**Not possible to extend to EF1 + PO allocation**

# EF1+PO for Mixed Items - an open problem





# EF1+PO for Mixed Items - an open problem



# Lexicographic Preferences State of Knowledge

	Only Goods	Only Chores	Mixed
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<b>MMS</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✓ P [Hosseini et al., 2023]
<b>MMS + PO</b>	✓ P [Hosseini et al., 2021]	✓ P [Hosseini et al., 2022]	✓ P [Hosseini et al., 2023]

# Conclusion

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It is worth studying restricted preferences

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It is worth studying restricted preferences

- Positive results in special cases

# Conclusion

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It is worth studying restricted preferences

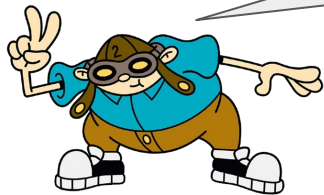
- Positive results in special cases
- Negative results for the general setting

# Conclusion

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It is worth studying restricted preferences

- Positive results in special cases
- Negative results for the general setting



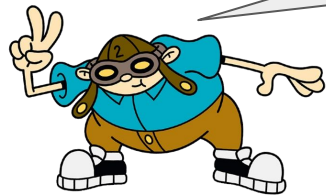
Thank  
you!

# Conclusion

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It is worth studying restricted preferences

- Positive results in special cases
- Negative results for the general setting



Happy  
Holidays!

