Participatory Budgeting with Cumulative Votes

Model

- projects: $P = \{p_1, \ldots, p_m\}$
- voters: $V = \{v_1, \ldots, v_n\}$; $v: P \rightarrow \mathbb{R}$; $v_j(p) \geq 0$; $\sum_{p \in P} v_j(p) = 1$
- costs: $c: P \rightarrow \mathbb{N}$
- budget limit: $L: \mathbb{N}$
- budgeting scenario: (P, V, c, L)

Greedy algorithms

- Greedy-by-Support (GS): $f_{GS}(p) = \sum_{j \in [n]} v_j(p)$.
- Greedy-by-Support-over-Cost (GSC): $f_{GSC}(p) = (1 / c(p)) \sum_{j \in [n]} (v_j(p) \cdot L/n)$.
- Greedy-by-Excess (GE): $f_{GSC}(p) = \sum_{j \in [n]} (v_j(p) \cdot L/n) - c(p)$.

Cumulative Single Transferable Vote (CSTV)

- Project-To-Fund Selection Procedure
- Excess Redistribution Procedure
- No-Eligible-Project Procedure
- Inclusive Maximality Postprocedure

Project-To-Fund Selection Procedure

Pick project to fund with greedy procedure

Excess Redistribution Procedure

- who needs votes to be transferred: tran(p) = $\{v_j \mid v_j(p) > 0$ and $\exists\, p' \notin S : v_j(p') > 0\}$.

- Transfer how much to transfer $$\frac{\gamma L}{n} \sum_{v_j \in \mathrm{tran}(p)} v_j(p) \; + \; \frac{L}{n} \sum_{v_j \notin \mathrm{tran}(p)} v_j(p) = c(p)\,.$$

- Transfer $(1 - \gamma) \cdot v_j(p)$ votes

No-Eligible-Project Procedure

- Elimination-with-Transfers (EwT)
- Minimal-Transfers (MT)

$$\frac{L}{n} \cdot \sum_{j:\, v_j(p)>0} \sum_{\ell=1}^{m} v_j(p_\ell) \geq c(p)$$

$$r = \mathrm{support}(\mathrm{p})/c(p)$$

$$v_j(p) := \min(\textstyle\sum_{\ell=1}^{m} v_j(p_\ell), \frac{v_j(p)}{r})$$

Inclusive Maximality Postprocedure

- Reverse Eliminations (RE)
- Acceptance of Undersupported Projects (AUP).

Selection of Variants

- EwT (i.e., GE + EwT + RE)
- EwTC (i.e., GSC + EwT + RE)
- MT (i.e., GE + MT + AUP)
- MTC (i.e., GSC + MT + AUP)

Axiomatic Properties

- Monotonicity Axioms
  - Splitting monotonicity
  - Merging monotonicity
  - Support monotonicity
- Proportional Representation
  - Weak Proportional Representation
  - Proportional Representation
  - Strong Proportional Representation

	GS	EwT	MT	GSC	EwTC	MTC
Splitting monotonicity	x	✓	✓	✓	x	x
Merging monotonicity	✓	x	x	x	x	x
Support monotonicity	x	x	x	x	x	x
Weak-PR	x	✓	✓	✓	✓	✓
PR	x	✓	✓	x	✓	✓
Strong-PR	x	x	✓	x	x	✓

Splitting monotonicity

	GS	EwT	MT	GSC	EwTC	MTC
Splitting monotonicity	x	✓	✓	✓	x	x
Merging monotonicity	✓	x	x	x	x	x
Support monotonicity	x	x	x	x	x	x
Weak-PR	x	✓	✓	✓	✓	✓
PR	x	✓	✓	x	✓	✓
Strong-PR	x	x	✓	x	x	✓

Definition 1 (Splitting monotonicity). *An aggregation method $\mathcal{R}$ satisfies* splitting monotonicity *if for each budgeting scenario $E = (P, V, c, L)$, for each funded project $p \in \mathcal{R}(E)$, and for each budgeting scenario E' which is formed by splitting project p into a set of projects P' with the same cost $c(p) = c(P')$, and such that for each voter v_i we have $v_i(P') = v_i(p)$, it holds that $\mathcal{R}(E') \cap P' \neq \emptyset$.*

Merging monotonicity

	GS	EwT	MT	GSC	EwTC	MTC
Splitting monotonicity	x	✓	✓	✓	x	x
Merging monotonicity	✓	x	x	x	x	x
Support monotonicity	x	x	x	x	x	x
Weak-PR	x	✓	✓	✓	✓	✓
PR	x	✓	✓	x	✓	✓
Strong-PR	x	x	✓	x	x	✓

Definition 2 (Merging monotonicity). *An aggregation method $\mathcal{R}$ satisfies* merging monotonicity *if for each budgeting scenario $E = (P, V, c, L)$, each $B' \subseteq \mathcal{R}(E)$, and for each scenario $E' = (P \setminus B' \cup \{b'\}, V, c', L)$ such that b' is a new project which costs $c(B')$ and such that for each voter v_i we have that $v_i(b') = \sum_{b \in B'} v_i(b)$, it holds that $b' \in \mathcal{R}(E')$.*

	GS	EwT	MT	GSC	EwTC	MTC
Splitting monotonicity	x	✓	✓	✓	x	x
Merging monotonicity	✓	x	x	x	x	x
Support monotonicity	x	x	x	x	x	x
Weak-PR	x	✓	✓	✓	✓	✓
PR	x	✓	✓	x	✓	✓
Strong-PR	x	x	✓	x	x	✓

Support monotonicity

Definition 3 (Support monotonicity). *An aggregation method* $\mathcal{R}$ *satisfies* support monotonicity *if for each budgeting scenario* $E = (P, V, c, L)$*, each project* $p \in \mathcal{R}(E)$*, and each budgeting scenario* $E' = (P, V', c, L)$ *such that* $|V \triangle V'| = 1$ *and for the single voter* $v \in V \triangle V'$ *and the single voter* $v' \in V' \triangle V$ *it holds that (1)* $v'(p) > v(p)$ *and (2) for each* $p' \neq p$, $v'(p') \leq v(p')$*, then* $p \in \mathcal{R}(E')$*.*

Weak Proportional Representation

	GS	EwT	MT	GSC	EwTC	MTC
Splitting monotonicity	x	✓	✓	✓	x	x
Merging monotonicity	✓	x	x	x	x	x
Support monotonicity	x	x	x	x	x	x
Weak-PR	x	✓	✓	✓	✓	✓
PR	x	✓	✓	x	✓	✓
Strong-PR	x	x	✓	x	x	✓

Definition 4 (Weak Proportional Representation). *An aggregation method $\mathcal{R}$ satisfies* Weak Proportional Representation (Weak-PR) *if for each budgeting scenario $E = (P, V, c, L)$, for each $\ell \in [L]$, each set $V' \subseteq V$ of voters with $|V'| \geq \ell n/L$, and each set $P' \subseteq P$ of projects with $c(P') \leq \ell$, there exist a scenario E' which differs from E only in the votes of the voters from V', such that $P' \subseteq \mathcal{R}(E')$.*

Proportional Representation

	GS	EwT	MT	GSC	EwTC	MTC
Splitting monotonicity	x	✓	✓	✓	x	x
Merging monotonicity	✓	x	x	x	x	x
Support monotonicity	x	x	x	x	x	x
Weak-PR	x	✓	✓	✓	✓	✓
PR	x	✓	✓	x	✓	✓
Strong-PR	x	x	✓	x	x	✓

Definition 5 (Proportional Representation). *An aggregation method $\mathcal{R}$ satisfies* Proportional Representation (PR) *if for each budgeting scenario $E = (V, P, c, L)$, each $\ell \in [L]$, each $V' \subseteq V$ with $|V'| \geq \ell n/L$, and each set $P' \subseteq P$ of projects with $c(P') \leq \ell$, it holds that: If all voters $v' \in V'$ support all projects in P', and no other projects, then $P' \subseteq \mathcal{R}(E)$.*

	GS	EwT	MT	GSC	EwTC	MTC
Splitting monotonicity	x	✓	✓	✓	x	x
Merging monotonicity	✓	x	x	x	x	x
Support monotonicity	x	x	x	x	x	x
Weak-PR	x	✓	✓	✓	✓	✓
PR	x	✓	✓	x	✓	✓
Strong-PR	x	x	✓	x	x	✓

Strong Proportional Representation

Definition 6 (Strong Proportional Representation). *An aggregation method* $\mathcal{R}$ *satisfies* Strong Proportional Representation (Strong-PR) *if for each scenario* $E = (P, V, c, L)$, *each* $\ell \in [L]$, *each* $V' \subseteq V$ *with* $|V'| \geq \ell n/L$, *and each* $P' \subseteq P$, *it holds that: If all voters* $v' \in V'$ *support all projects in* P', *and not any other project, then either* $P' \subseteq \mathcal{R}(E)$ *or for each* $p \in P' \setminus \mathcal{R}(E)$ *we have that* $c(p) + c(P' \cap \mathcal{R}(E)) > \ell$.

Experimental Evaluation

Voter Satisfaction (VS): fraction of support of a voter which went on funded projects (formally, for a winning bundle B, the voter satisfaction of voter v is $\sum_{p \in B} v(p)$).

Anger Ratio (AR): the fraction of voters who are ignored in the election (formally, $|\{v : \sum_{p \in B} v(p) = 0\}|/|V|$).

Voter Satisfaction with Approval Votes (*VS): instead of using Equation (1), for each $v_j \in V$ and $p \in S_j$ we set $v_j(p) = 0.1$. This corresponds to using approval ballots (a voter supports each of her supported projects equally).

Simulations

(a) Simulation Scenario 1

Rule	VS	*VS	suburbs	AR	AC
target	100%	100%	40%	0.0	-
GS	23.0 %	20.0%	10.9 %	25.4%	45k
EwT	25.0 %	23.9%	21.6 %	2.1%	25 k
MT	22.6 %	22.2%	31.8 %	3.5%	24 k
GSC	27.6 %	25.4%	15.2 %	11.0%	29 k
EwTC	25.9 %	24.1%	16.7 %	2.8%	27 k
MTC	25.7 %	23.8%	22.9 %	2.9%	28 k

(b) Simulation Scenario 2

Rule	VS	*VS	FoEP	AR	AC
target	100%	100%	50%	0.0%	-
GS	21.2 %	18.9%	65%	27.1%	45 k
EwT	24.4 %	23.1%	32%	12.1%	26 k
MT	22.8 %	22.5%	22%	19.7%	25 k
GSC	27.9 %	26.3%	7%	39.0%	25 k
EwTC	24.5 %	22.7%	41%	8.9%	29 k
MTC	24.3 %	22.9%	40%	11.2%	29 k

(c) Warsaw Instance

Rule	VS	AR	AC
WM	66%	5.1%	860 k
GS	67%	4.6%	804 k
EwT	80%	2.6%	295 k
MT	80%	2.5%	294 k
GSC	81%	2.7%	324 k
EwTC	81%	2.8%	319 k
MTC	81%	2.7%	319 k

Based on:
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