A Generalised Theory of Proportionality in Collective Decision Making

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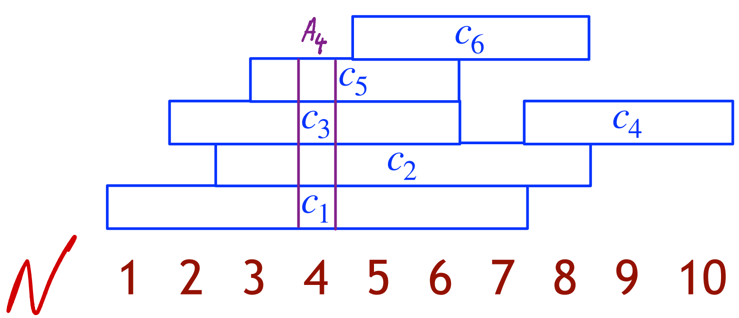
The model

1. A set of *candidates* or *projects* $C = \{c_1, c_2, \ldots, c_m\}$.
2. A set of voters $N = \{1, 2, \ldots, n\}$.
   A_i : the set of projects approved by voter i.

$A_4 = \{c_1, c_2, c_3, c_5\}$



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3. The goal is to select a subset of candidates.

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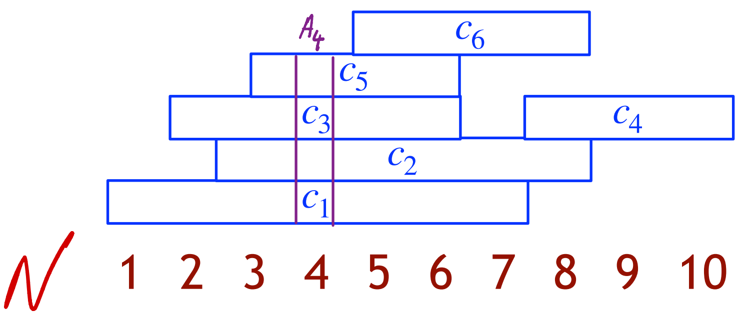

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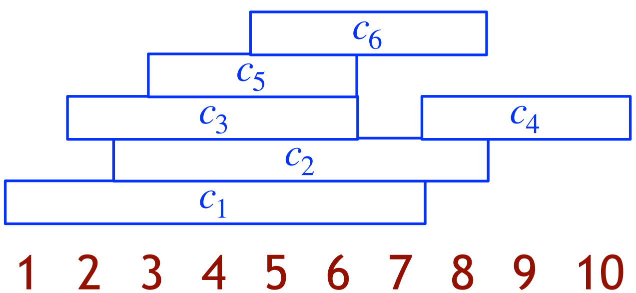
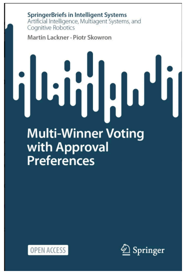


A subset of a given size k

↓

committee elections



The model

1. A set of *candidates* or *projects* $C = \{c_1, c_2, \ldots, c_m\}$. with weights
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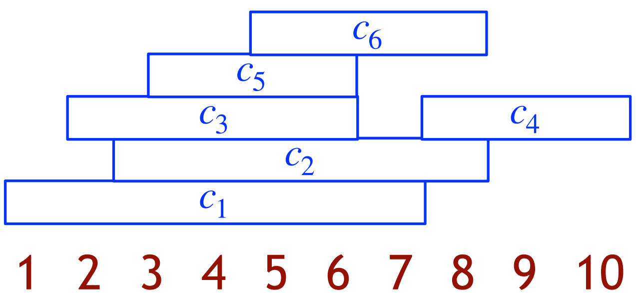

A subset of candidates with the total cost not exceeding the budget value b.

↓

participatory budgeting

S. Rey, J. Maly: The (Computational) Social Choice Take on Indivisible Participatory Budgeting, 2023.

The model

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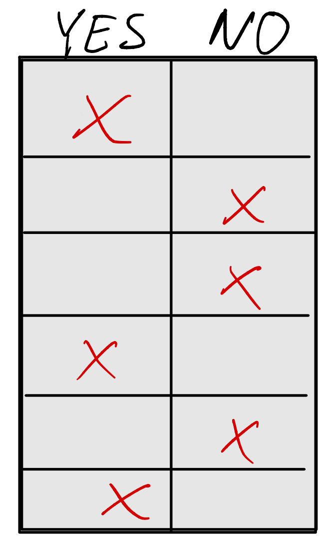
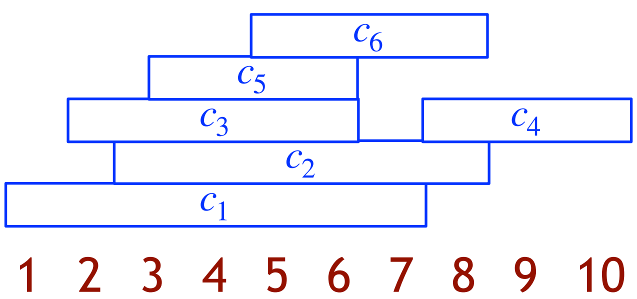




The candidates are grouped into pairs and foreach pair we need to select one.

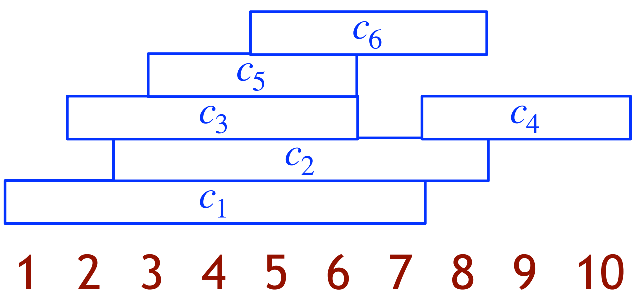
public decisions

V. Conitzer, R. Freeman, and N. Shah. Fair public decision making. EC-2017.

R. Freeman, A. Kahng, and D. M. Pennock. Proportionality in approval-based elections with a variable number of winners. IJCAI-2020.

The model

1. A set of *candidates* or *projects* $C_1 = \{c_1, c_2, c_3\}$ $C_2 = \{c_4, c_5, c_6\}$ $C_3 = \{c_7, c_8, c_9\}$ ⋮
2. A set of voters $N = \{1, 2, \ldots, n\}$.
   A_i : the set of projects approved by voter i.
3. The goal is to select a subset of candidates. e.g. $|C_i \cap W| = 1$



A subset of a given size k with **diversity constraints.**

L. E. Celis, L. Huang, and N. K. Vishnoi. Multiwinner voting with fairness constraints. IJCAI-2018.

R. Bredereck, P. Faliszewski, A. Igarashi, M. Lackner, and P. Skowron. Multiwinner elections with diversity constraints. AAAI-2018.

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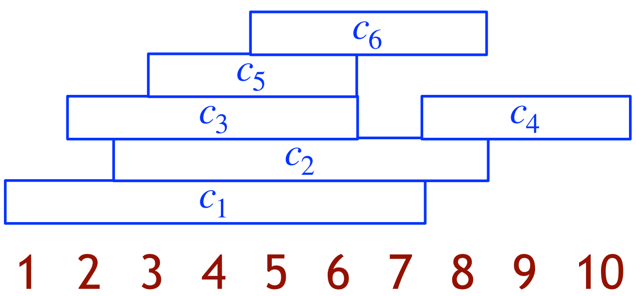

Ranking candidates

For each pair, c_1 and c_2, we introduce an auxiliary candidate $c_{1,2}$, whose selecting corresponds to ranking c_1 before c_2.

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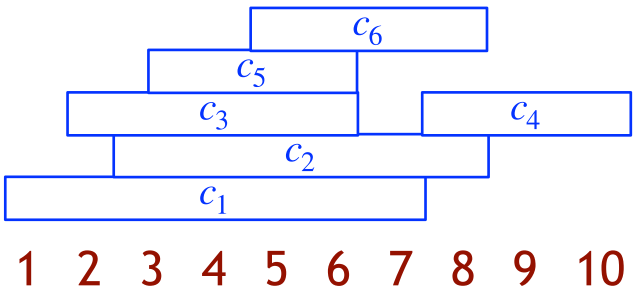

Committee elections with negative votes

For each c we introduce an auxiliary candidate $\bar{c}$, whose selecting corresponds to not selecting c.

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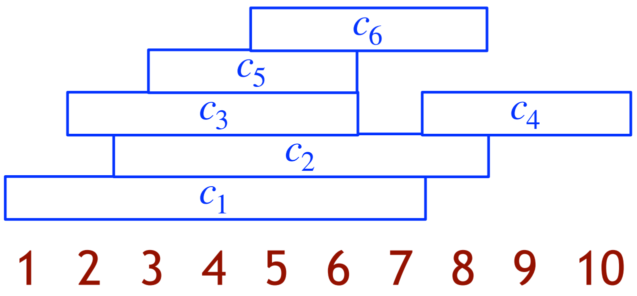

Our general model

We are given a nonempty **family of feasible sets** $\mathcal{F} \subseteq 2^C$.

($\mathcal{F}$ is closed under inclusions)

Proportionality in the general model

For committee elections:

An ℓ-cohesive group: a group of voters $S \subseteq N$ is cohesive if

(1) $|S| \geq \ell \cdot n/k$, and (2) $\left|\bigcap_{i \in S} A_i\right| \geq \ell$.

Proportional size

all approve ℓ candidates

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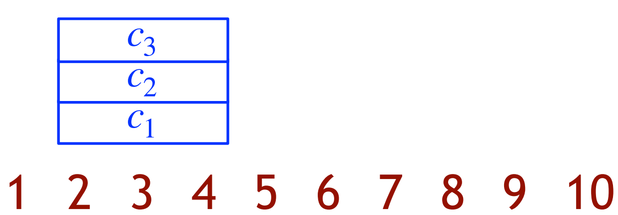
Extended Justified Representation (EJR): an outcome W satisfies extended justified representation if for each ℓ-cohesive group of voters S it holds that:

there exists $i \in S$ such that $|A_i \cap W| \geq \ell$

$k = 10$



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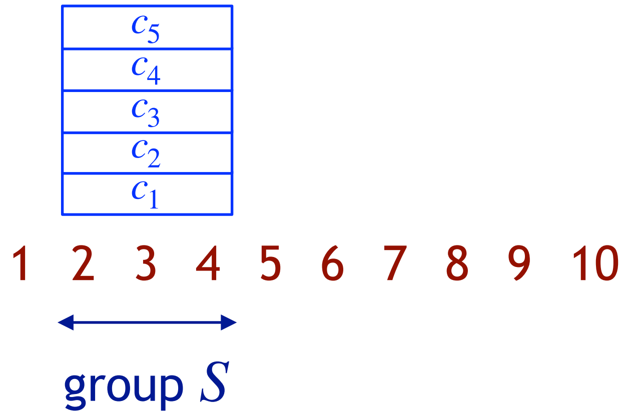
The challange is how to properly define

ℓ-cohesiveness in the general model.

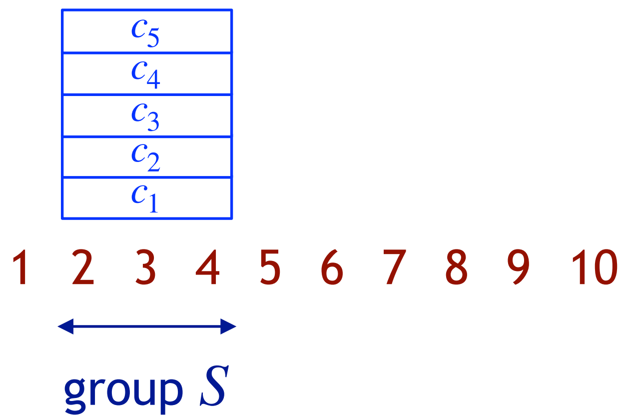
Proportionality in the general model



Group S agrees on some ℓ candidates. Do they deserve ℓ candidates?

↳ having proportional size

Proportionality in the general model

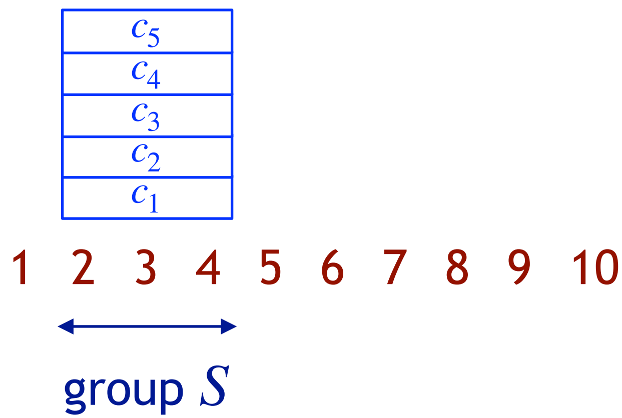


Group S agrees on some ℓ candidates. Do they deserve ℓ candidates?

Selecting ℓ candidates supported by S might use "too many feasibility slots" and deprive the other voters, $N \setminus S$, from the set T that they like.

even while S has proportional size

Proportionality in the general model



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This would be unfair if:

$$\frac{\ell}{|S|} > \frac{|T|}{n - |S|}.$$

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rearanged

$$\frac{|S|}{n} > \frac{\ell}{|T| + \ell}$$

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A group of voters $S \subseteq N$ is ℓ-cohesive if for each feasible set $T \in \mathcal{F}$ at least one of the following conditions hold:

case $T = \emptyset$

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   $X \in \mathcal{F}$

2. Or

$$\frac{|S|}{n} > \frac{\ell}{|T| + \ell}$$

not possible

$\Rightarrow$ All voters in S approve same ℓ candidates

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"Proportional size in general model"

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To get EJR in the definition of ℓ-cohesiveness we look only at $T \subseteq W$.

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Example (committee elections):

Group S of 30% of voters, who approve 3 candidates; $k = 10$.

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1. If $|T| \leq 7$ then we can add these 3 candidates and the set is feasible.

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Example (committee elections):

Group S of 30% of voters, who approve 3 candidates; $k = 10$.

1. If $|T| \leq 7$ then we can add these 3 candidates and the set is feasible.
2. If $|T| > 7$ then

$$\frac{|S|}{n} = 0.3 > \frac{3}{3 + |T|}$$

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$$\frac{|S|}{n} > \frac{\ell}{|T| + \ell}$$

YES	NO
X	
	X
	X
X	
	X
X	

Example: Public decisions with p yes/no issues.

Group of 30% of voters, who approve jointly p decisions. (Have the same opinion on them);

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YES	NO
X	
	X
	X
X	
	X
X	

Example: Public decisions with p yes/no issues.

Group of 30% of voters, who approve jointly p decisions. (Have the same opinion on them);

If $|T| \leq p - \lfloor 0.3p \rfloor \Rightarrow \exists \lfloor 0.3p \rfloor$ decisions D s.t. $T \cup D \in \mathcal{F}$

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$$0.3 \simeq \frac{|S|}{n} > \frac{\ell}{|T| + \ell} = \frac{\lfloor 0.3p \rfloor}{p - \lfloor 0.3p \rfloor + \varepsilon + \lfloor 0.3p \rfloor}$$

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Else

YES	NO
X	
	X
	X
X	
	X
X	

No guarantee for such a group if p separate elections!

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Example (committee elections with 50% of men and 50% of women):

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Example (committee elections with 50% of men and 50% of women):
Group S of 30% of voters, who approve 100 woman candidates; $k = 100$.

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$$\frac{3}{10} = \frac{|S|}{n} > \frac{\ell}{|T| + \ell} = \frac{15}{35+15}$$

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The group S is entitled to 30% of 50 that is to 15 candidates.

is 15-cohesive

Proportionality in the general model

Related work:

I.-A. Mavrov, K. Munagala, and Y. Shen. Fair multiwinner elections with allocation constraints. EC-2023

This paper introduces Restrained EJR. However,

1. In this example it provides no guarantees to the group S.
2. Is implied by our definition of EJR.
3. In general might contradict Pareto-Optimality

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This definition of Base EJR (and so EJR) implies:

1. EJR in the model of committee elections.

H. Aziz, M. Brill, V. Conitzer, E. Elkind, R. Freeman, and T. Walsh. Justified representation in approval-based committee voting. Social Choice and Welfare, 2017.

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2. **Strong EJR** in the model of sequential decision making.

   N. Chandak, S. Goel, and D. Peters. Proportional aggregation of preferences for sequential decision making. 2023.

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2. **Strong EJR** in the model of sequential decision making.

N. Chandak, S. Goel, and D. Peters. Proportional aggregation of preferences for sequential decision making. 2023.

3. **Proportionality for cohesive groups** in the model of public decisions.

P. Skowron and A. Górecki. Proportional public decisions. AAAI-2022.

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We can use this idea to extend other notions of propotionality.

(Base) EJR

↓

(Base) PJR

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↓

(Base) PJR

↓

(Base) JR

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We can use this idea to extend other notions of propotionality.

(Base) EJR → (for matroids) → (Base) proportionality degree of $\frac{\ell - 1}{2}$

(Base) EJR

↓

(Base) PJR

↓

(Base) JR

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(Base) EJR ← (Base) FJR ← (Base) core

(Base) EJR ↓ (Base) PJR ↓ (Base) JR

(Base) EJR → for matroids → (Base) proportionality degree of $\dfrac{\ell - 1}{2}$

Why our definition is appealing?

1. It implies the strongest known JR-notions in the more specific models.

This definition of Base EJR (and so EJR) implies:

1. **EJR** in the model of committee elections.

   H. Aziz, M. Brill, V. Conitzer, E. Elkind, R. Freeman, and T. Walsh. Justified representation in approval-based committee voting. Social Choice and Welfare. 2017.

2. **Strong EJR** in the model of sequential decision making.

   N. Chandak, S. Goel, and D. Peters. Proportional aggregation of preferences for sequential decision making. 2023.

3. **Proportionality for cohesive groups** in the model of public decisions.

   P. Skowron and A. Górecki. Proportional public decisions. AAAI-2022.

Why our definition is appealing?

2. **Theorem**: an outcome satisfying Base FJR always exists!

Why our definition is appealing?

3. **Theorem**: PAV satisfies (Base) EJR if and only if $\mathscr{F}$ is a matroid.

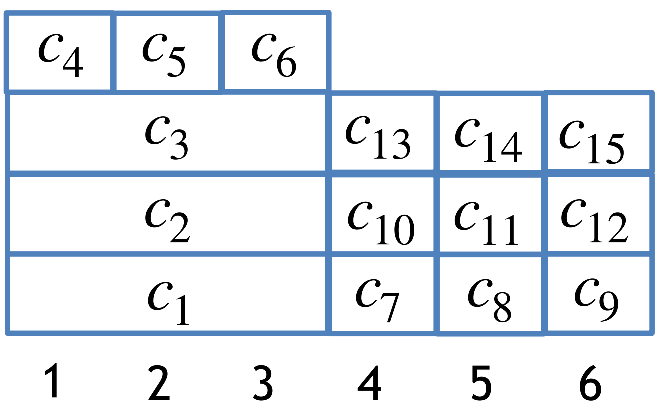
Proportional Approval Voting (PAV): select an outcome W that maximizes :

$$\sum_{i \in N} \mathrm{H}(|A_i \cap W|) \qquad \text{where} \qquad \mathrm{H}(z) = \sum_{j=1}^{z} \frac{1}{j}$$

Why our definition is appealing?

4. **Theorem**: Phragmen's Rule satisfies (Base) PJR if and only if $\mathscr{F}$ is a matroid.

$$k = 12$$



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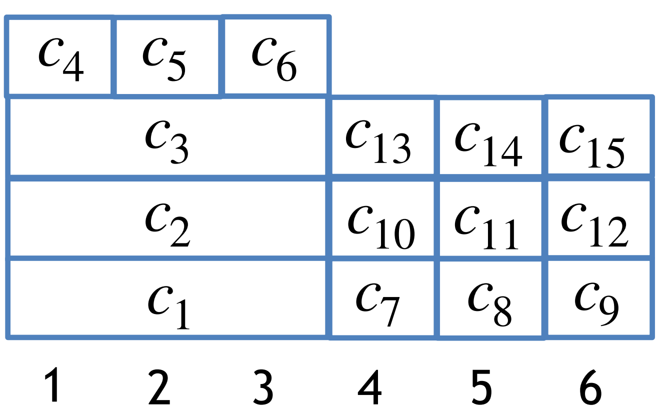

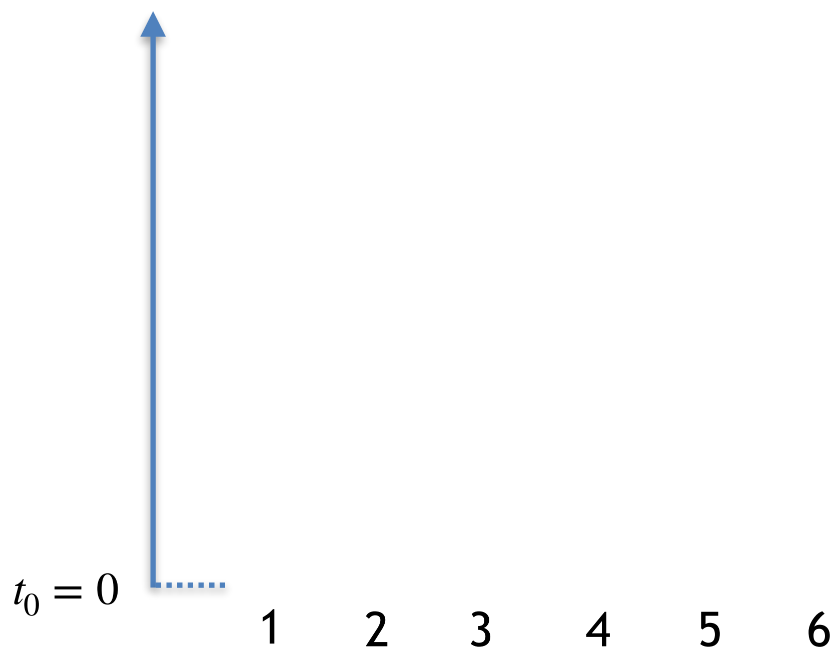

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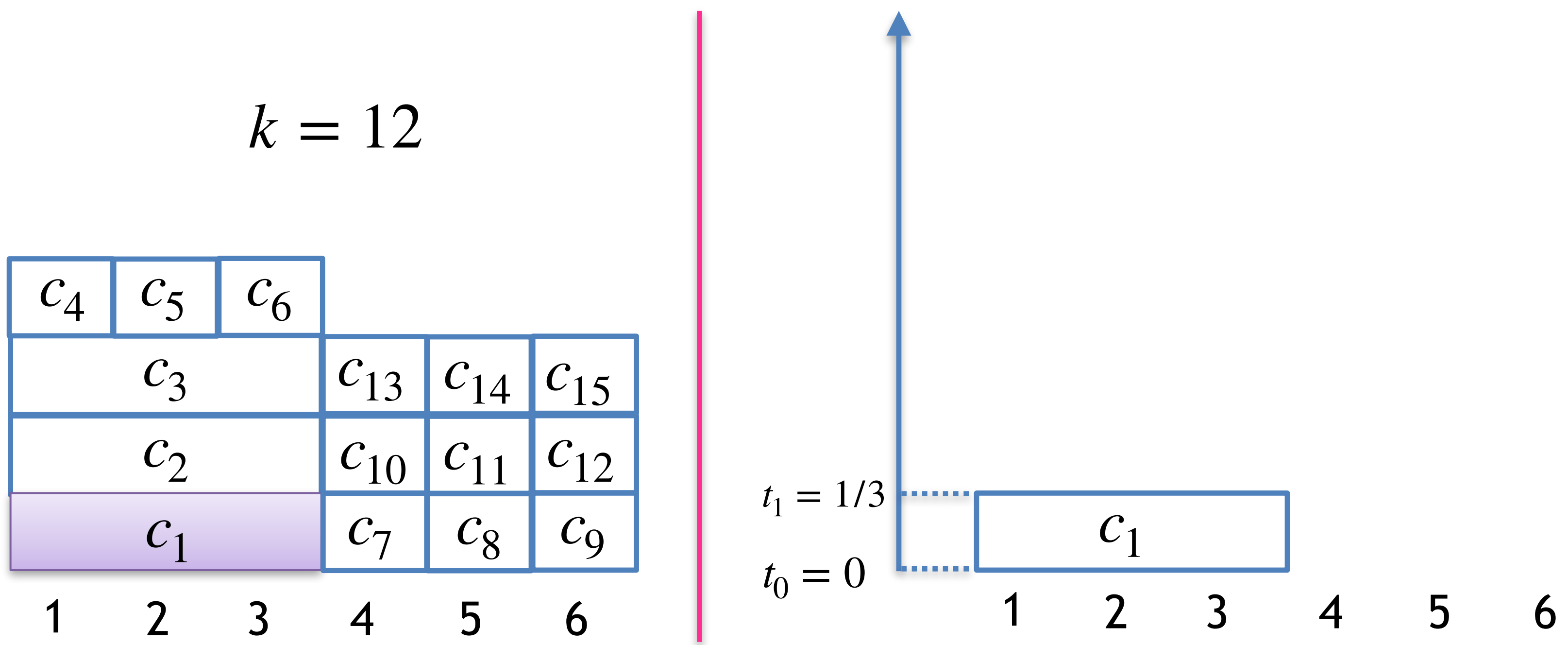

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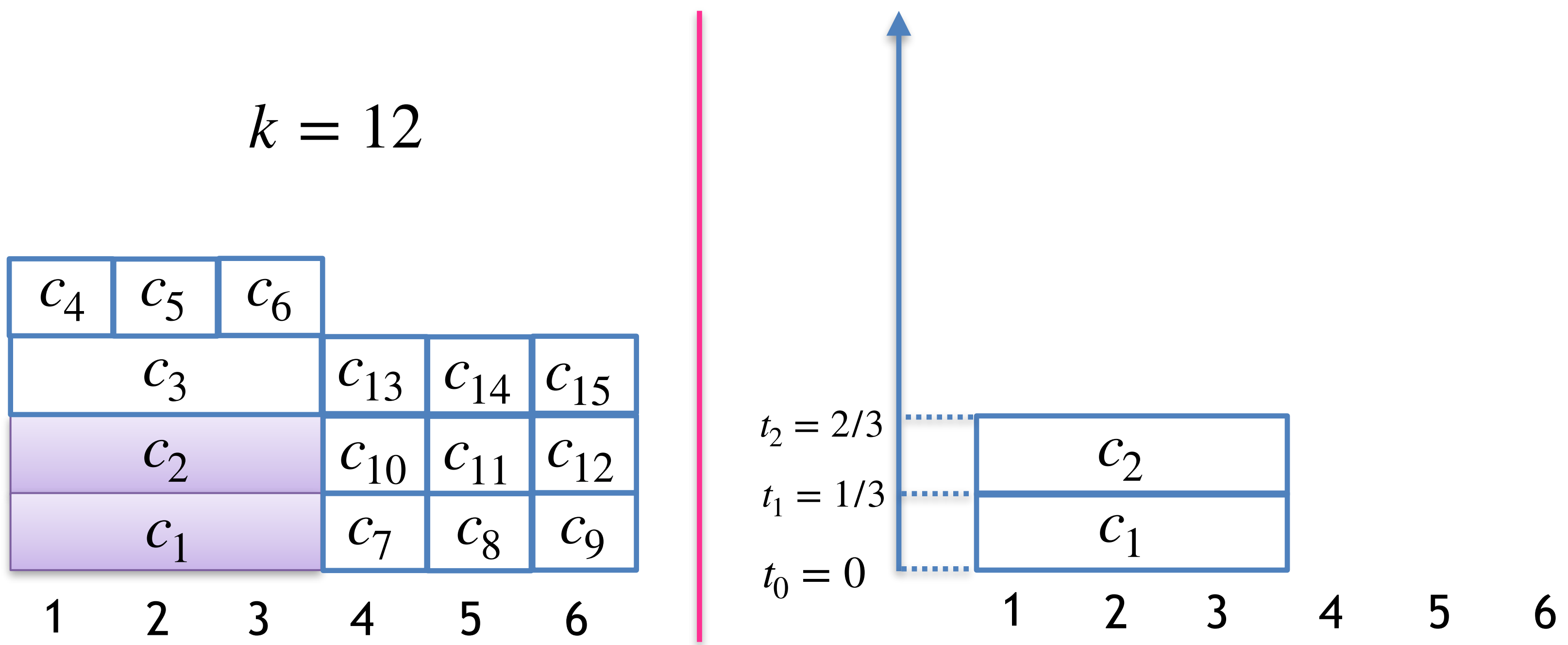

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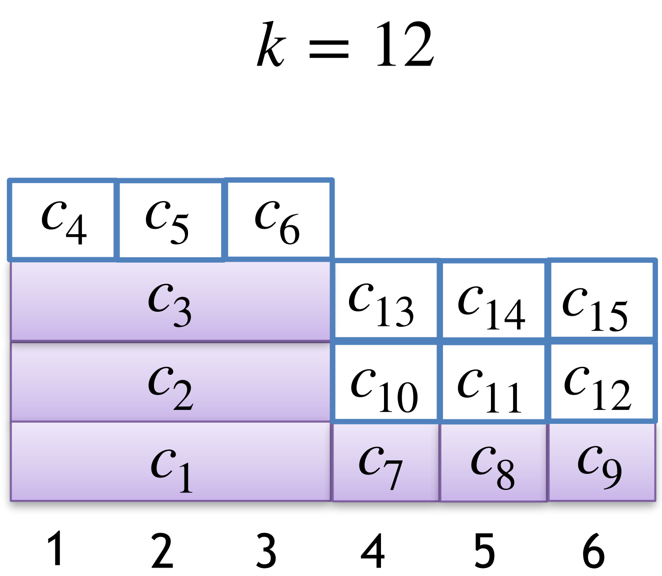

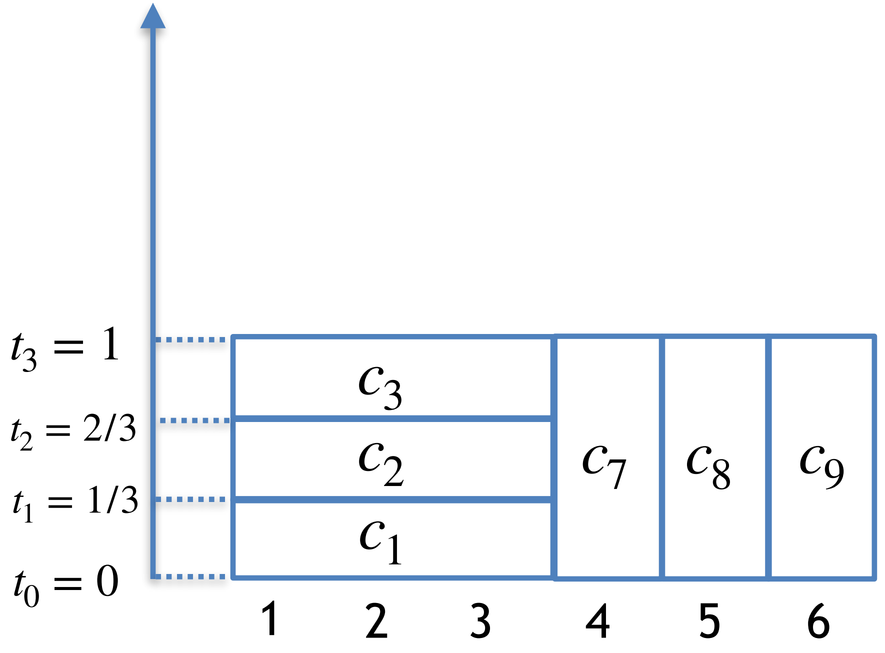

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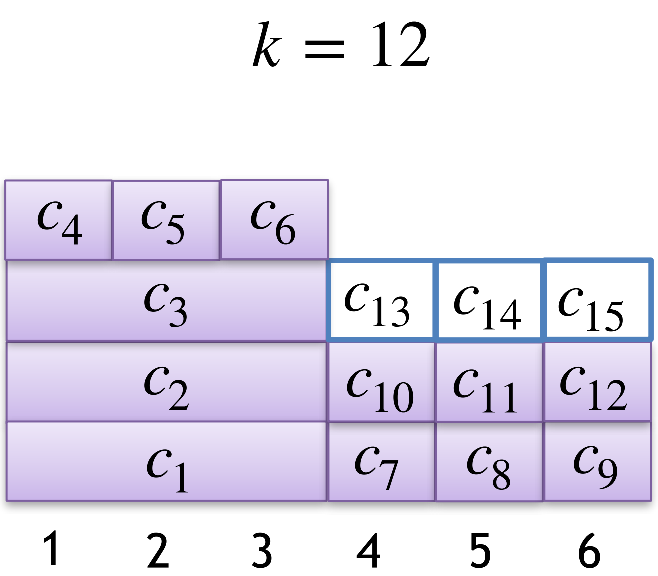

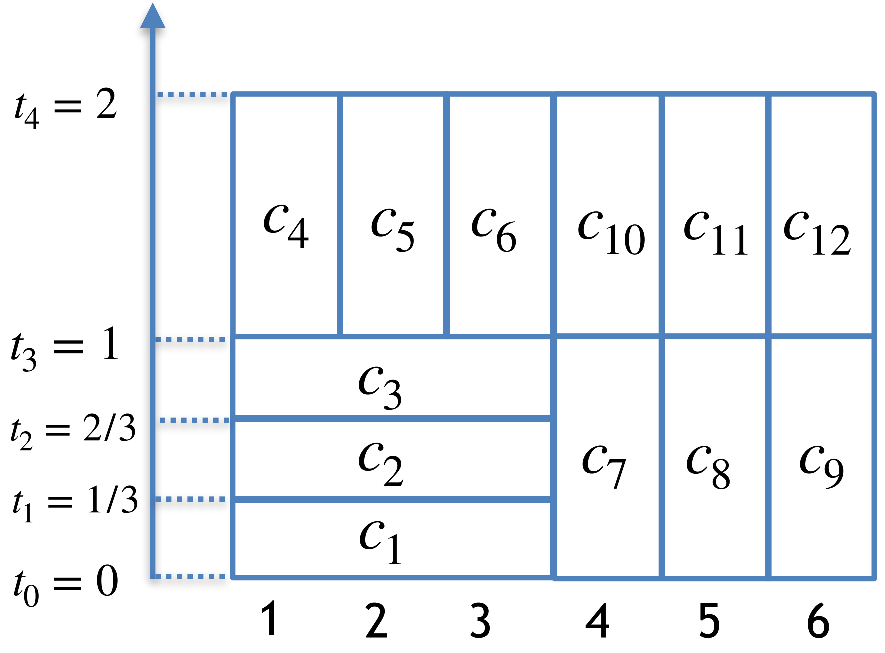

Why our definition is appealing?

5. **Theorem**: Phragmen's Rule has the proportionality degree of $\frac{\ell - 1}{2}$ if $\mathscr{F}$ is a matroid.

$\Updownarrow$

On average the utility of a voter in S is $\frac{\ell-1}{2}$ for each S that is ℓ-cohesive.

A group of voters $S \subseteq N$ is ℓ-cohesive if for each feasible set $T \in \mathscr{F}$ at least one of the following conditions hold:

1. Either there exists $X \subseteq \bigcap_{i \in S} A_i$ with $|X| = \ell$ s.t. $X \cup T \in \mathscr{F}$,
2. Or

$$\frac{|S|}{n} > \frac{\ell}{|T| + \ell}$$

Why our definition is appealing?

6. **Theorem**: Stable priceability implies EJR if $\mathcal{F}$ is a matroid.

D. Peters, G. Pierczyński, N. Shah, and P. Skowron. Market-based explanations of collective decisions. I AAAI-2021.

Why our definition is appealing?

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2. **Theorem**: an outcome satisfying Base FJR always exists!
3. **Theorem**: PAV satisfies (Base) EJR if and only if $\mathscr{F}$ is a matroid.
4. **Theorem**: Phragmen's Rule has the proportionality degree of $\dfrac{\ell - 1}{2}$ if $\mathscr{F}$ is a matroid.
5. **Theorem**: Phragmen's Rule satisfies (Base) PJR if and only if $\mathscr{F}$ is a matroid.
6. **Theorem**: Stable priceability implies EJR if $\mathscr{F}$ is a matroid.

Summary

The model is pretty well understood for matroid constrains.

A group of voters $S \subseteq N$ is ℓ-cohesive if for each feasible set $T \in \mathscr{F}$ at least one of the following conditions hold:

1. Either there exists $X \subseteq \bigcap_{i \in S} A_i$ with $|X| \geq \ell$ s.t. $X \cup T \in \mathscr{F}$,
2. Or

$$\frac{|S|}{n} > \frac{\ell}{|T| + \ell}$$

Summary

The model is pretty well understood for matroid constrains.

When the candidates have weights

A group of voters $S \subseteq N$ is (α, β)-cohesive if for each feasible set $T \in \mathscr{F}$ at least one of the following conditions hold:

1. Either there exists $X \subseteq \bigcap_{i \in S} A_i$ with $\text{weight}(X) \leq \alpha$ and $|X| \geq \beta$ s.t. $X \cup T \in \mathscr{F}$,
2. Or

$$\frac{|S|}{n} > \frac{\alpha}{\text{weight}(T) + \alpha}$$

Summary

The model is pretty well understood for matroid constrains.

When the candidates have weights

Our results:

1. Phragmen's Rule provides a good approximation of PJR, yet it may fail PJR.
2. Stable-priceability implies a good approximation of EJR.

No matroid assumption!

Summary

Future directions

- Adapt MES
- More generic utilities
- Does FJR always exists?
- Core

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Thank you!