Selecting the Most Conflicting Pair of Candidates

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WKRECAI
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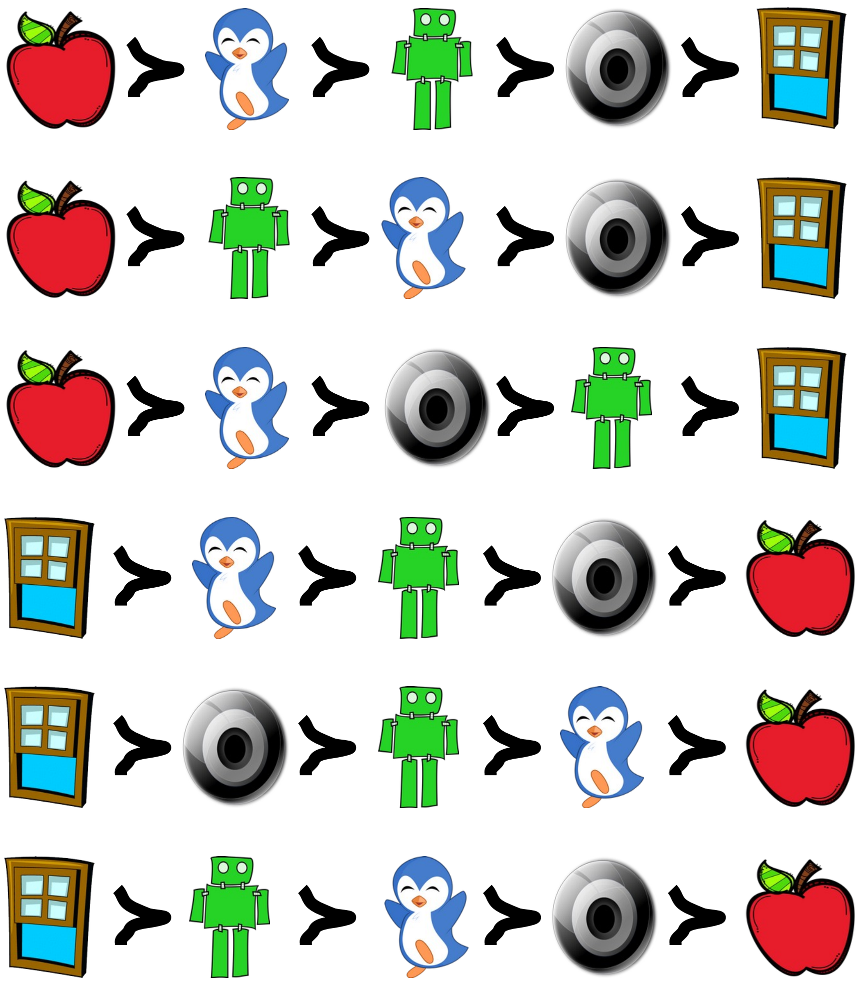
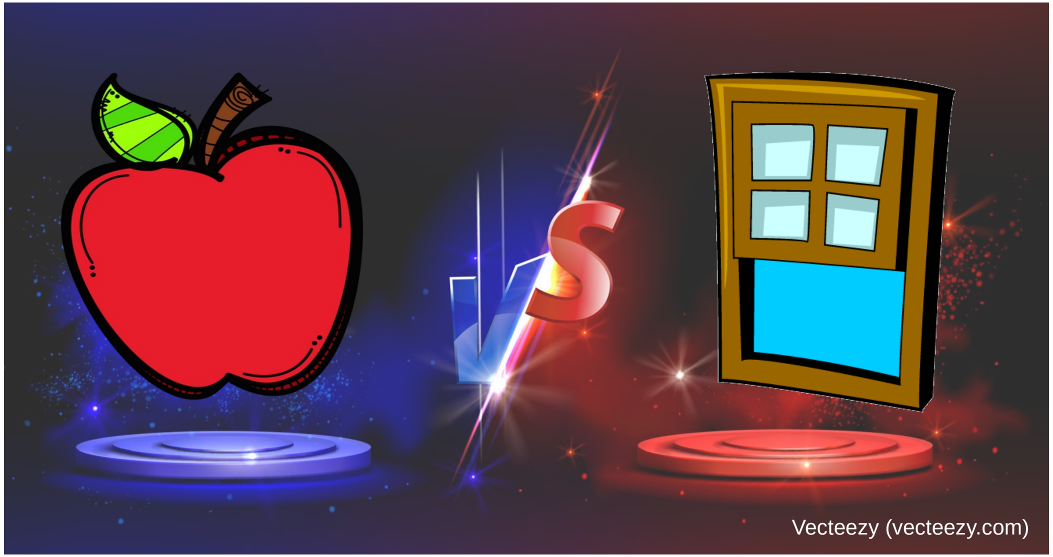
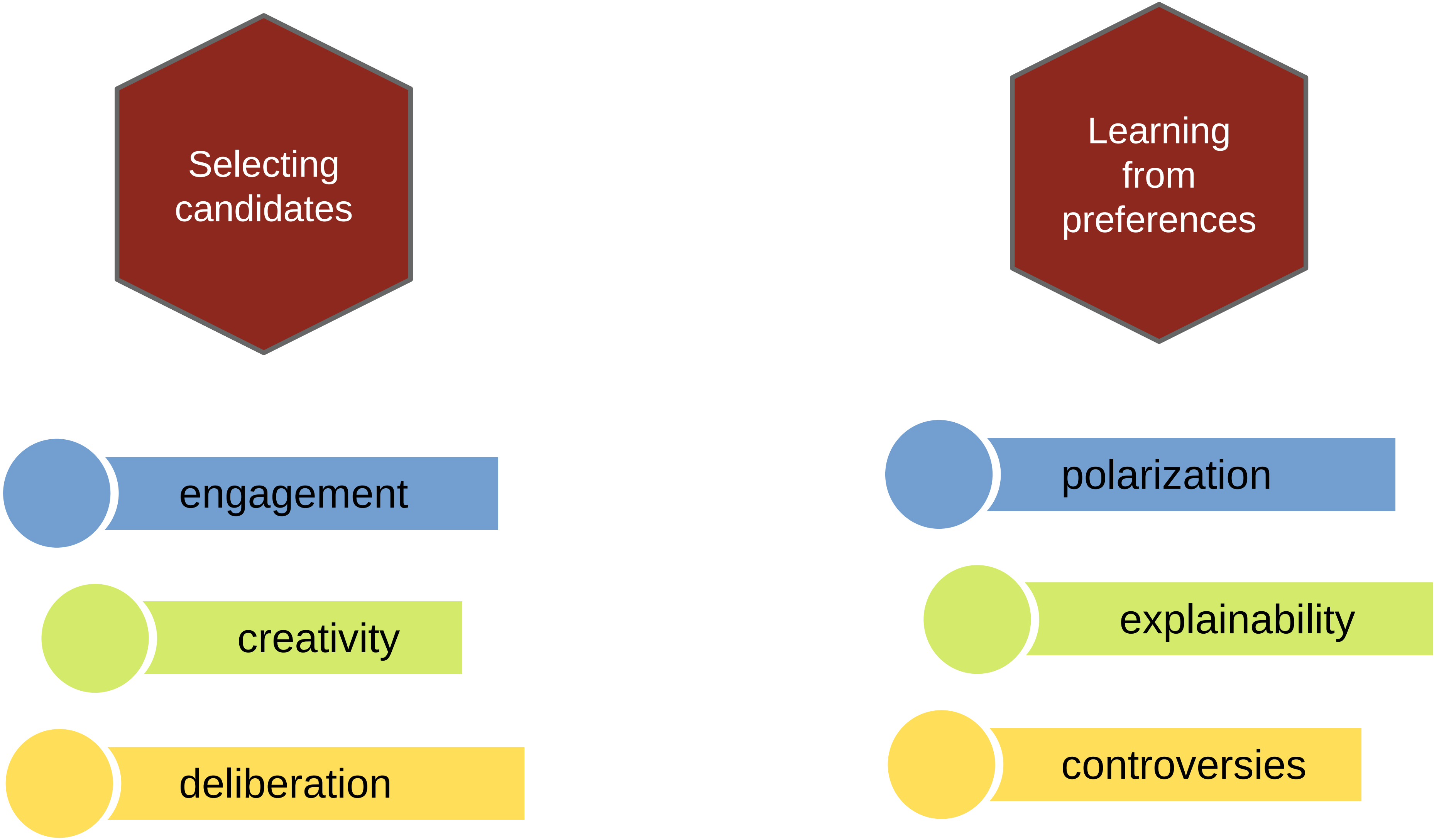
setting the stage

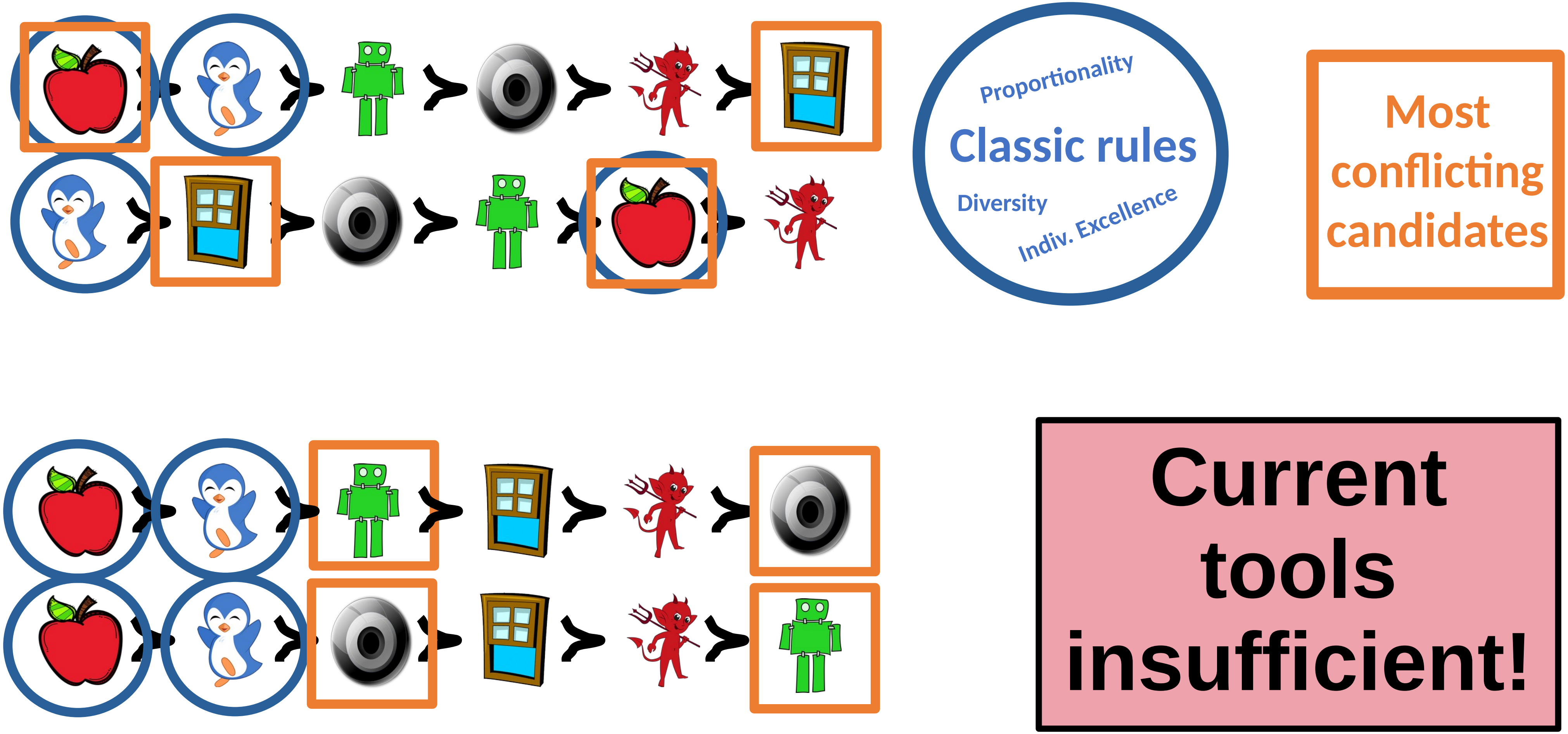



Vecteezy (vecteezy.com)

for what?

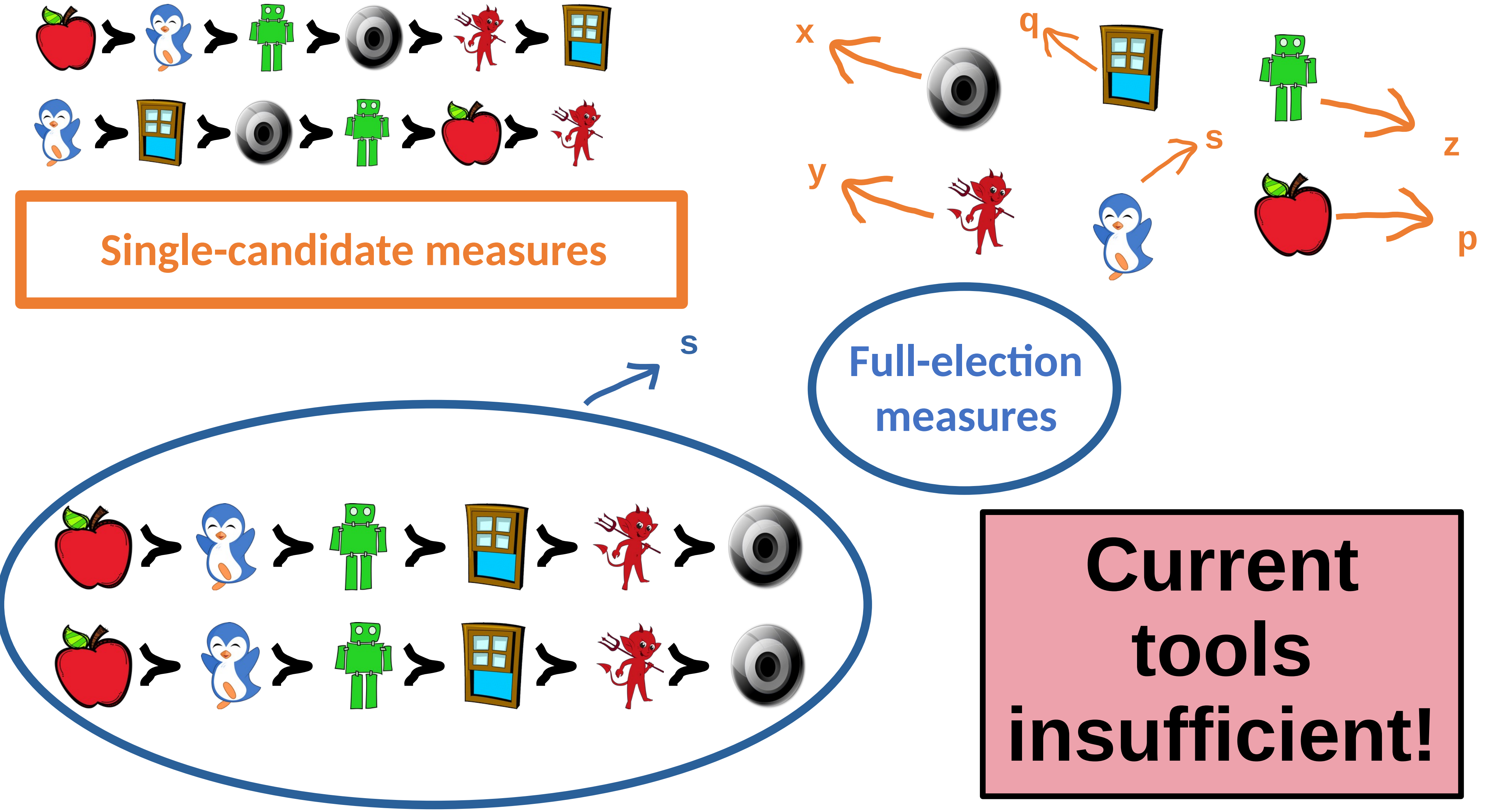

Selecting candidates
engagement
creativity
deliberation
Learning from preferences
polarization
explainability
controversies

selecting: state of the art



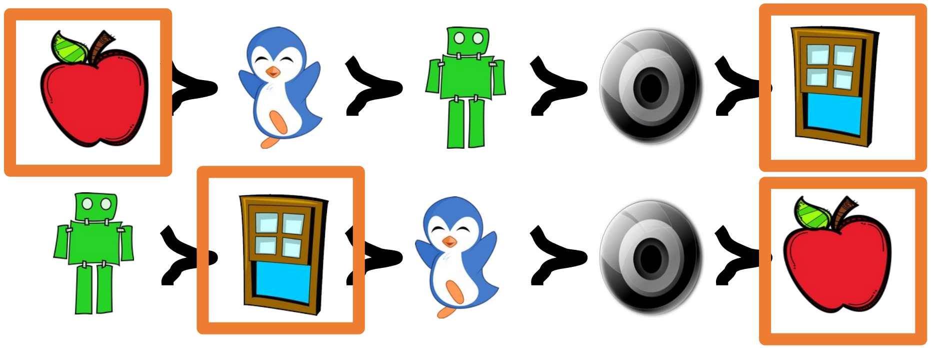
Proportionality
Classic rules
Diversity
Indiv. Excellence
Most conflicting candidates
Current tools insufficient!

preference insights: state of the art



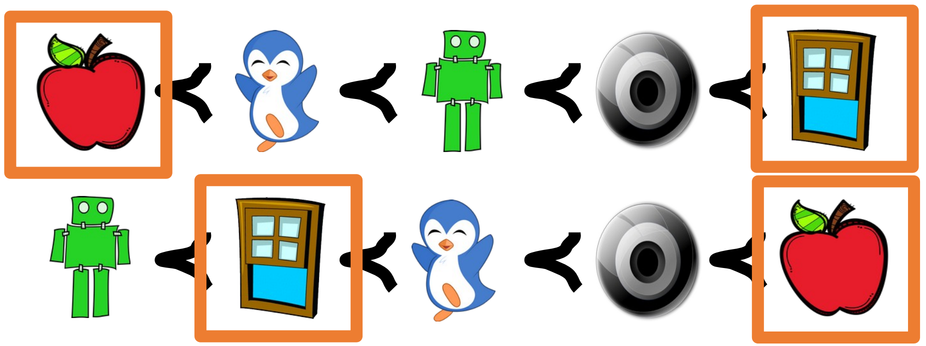
Single-candidate measures
x
q
z
s
y
p
s
Full-election measures
Current tools insufficient!

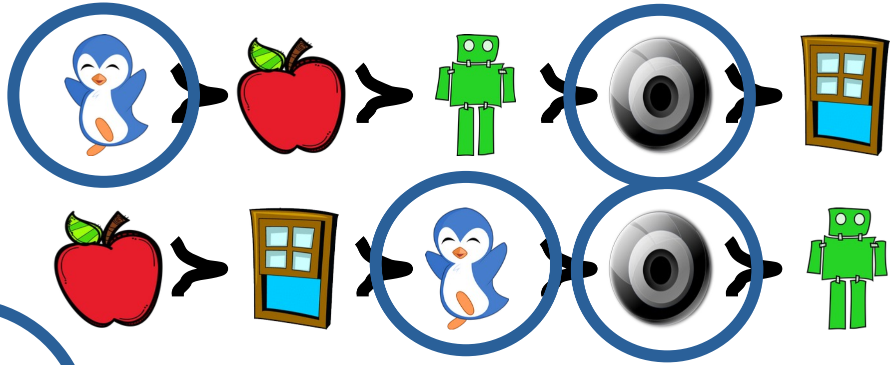
must-have properties

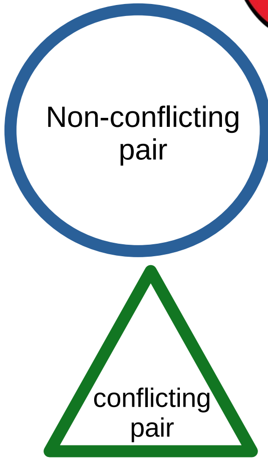


Reverse-stability

Reversing all orders does not change winners

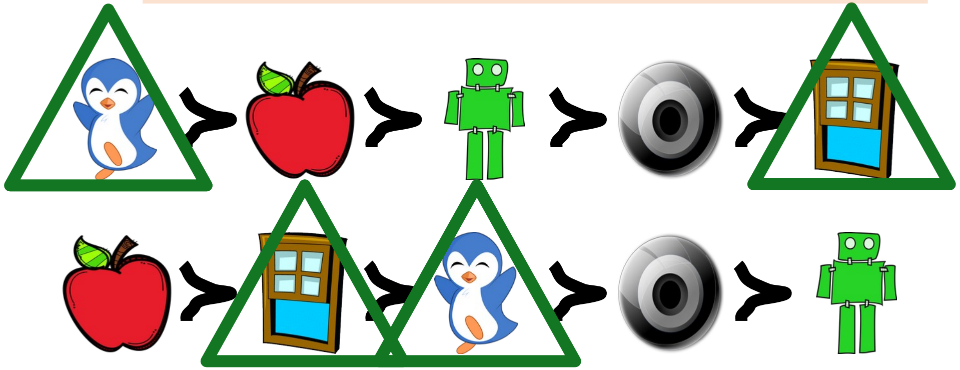






Conflict consistency

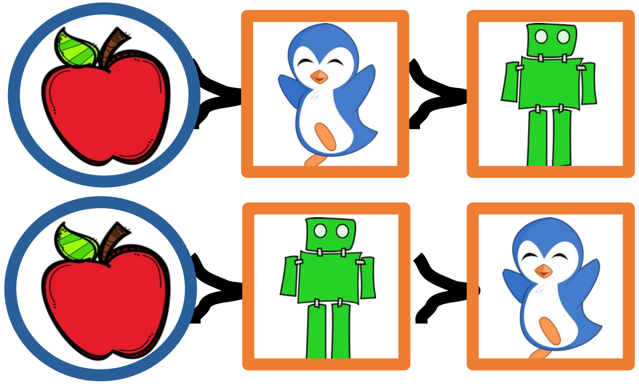
Non-conflicting pair does not win (if it exists)



Conflict consistency

Unanimity

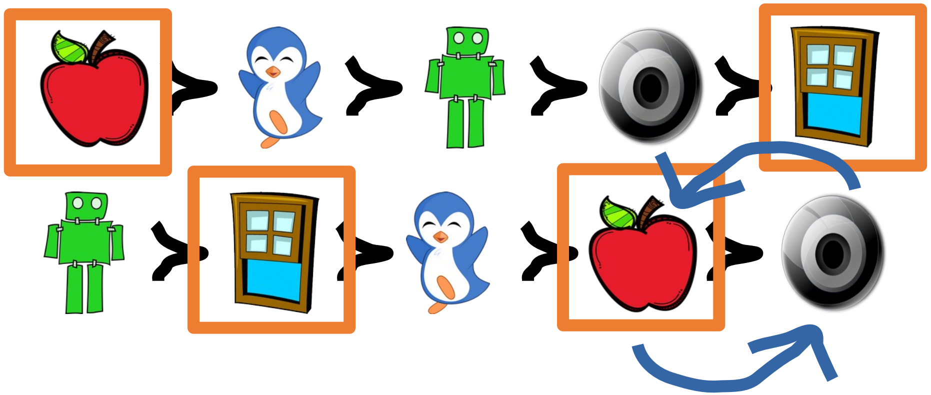
Must win
due to
conflict consistency



Must win due
to unanimity

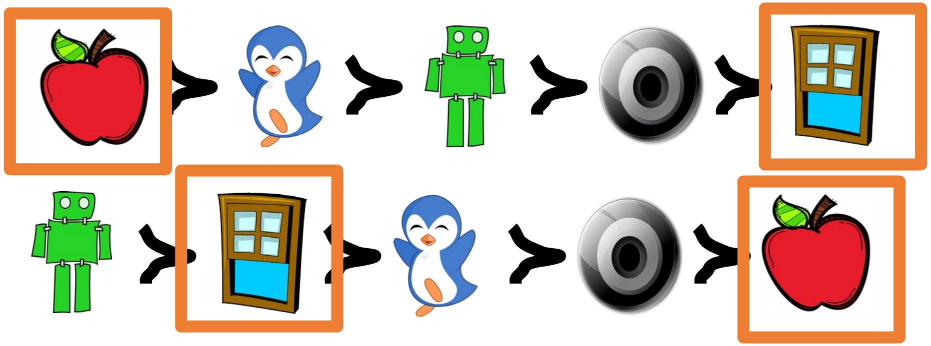
Conflict consistency and unanimity are contradicting each other!

nice-to-have properties



Conflict monotonicity

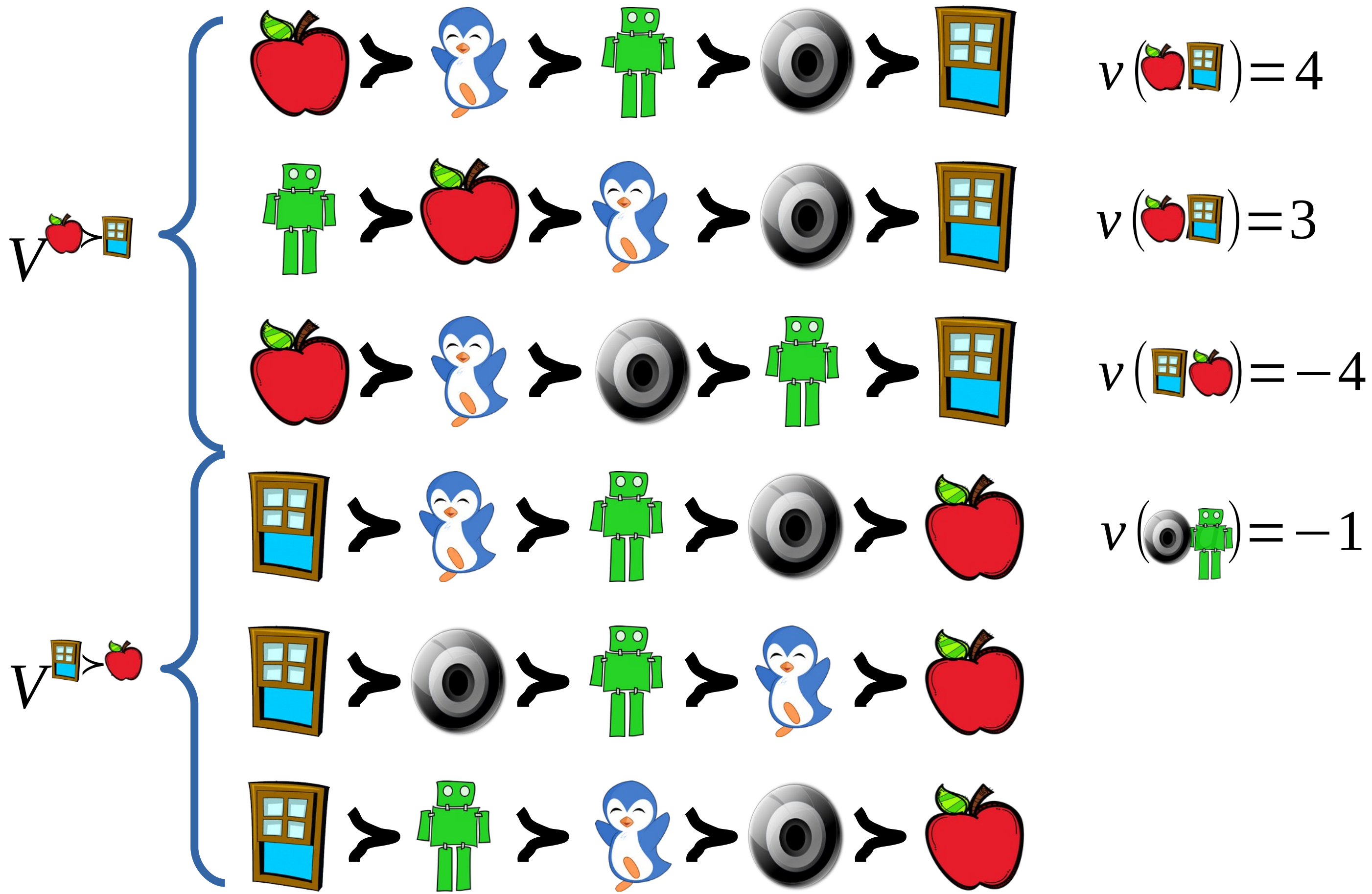
Increasing conflict for a selected pair does not make it lose



Math notation warning!

$V^{a \succ b}$ voters preferring a to b

$v(ab)$ "directed" positions difference between a and b



Matching-domination of pairs (informally)

Pair {A,B} dominates pair {C,D} if voters can be matched such that for each matched pair the conflict between A and B is at least that between C and D; with these inequality being strong for at least one pair. Each matched pair of voters has the same preference towards {A,B} and {C,D}.

	A>B $v(AB)$	C>D $v(CD)$
A>B>C>D	1	1
A>C>D>B	3	1
A>C>B>D	2	2
D>C>A>B	1	-1

A>B		C>D
3	$\geq$	2
2	$\geq$	1
1	$\geq$	1
1	$\geq$	1

A>B is dominating D>C

	B>A $v(BA)$	D>C $v(DC)$
B>D>A>C	2	2
B>C>D>A	3	-1

B>A		D>C
3	$\geq$	2
2	$\geq$	1

Matching domination

Matching-dominated pairs are never selected!

nobody’s perfect

Theorem: **Conflict consistency, matching domination**, and **conflict monotonicity** are **incompatible.**

Proof:

$a \succ b \succ c \succ d$
$b \succ a \succ d \succ c$

Only (a,b) or (c,d) can win

Conflict Consistency

Assume (a,b) is wins

$a \succ b \succ c \succ d$
$b \succ d \succ c \succ a$

(a,b) should still win

Conflict Monotonicity

$a \succ b$: $(1, -3)$
$a \succ d$: $(-3, 2)$

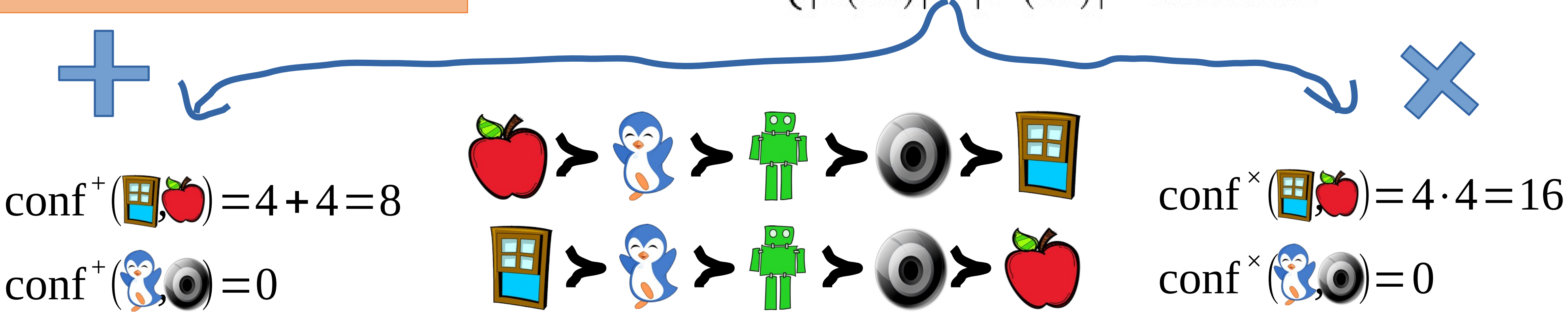
(a,d) dominates (a,b),
thus (a,b) cannot win

Matching Domination

getting the most conflicting pair

Conflict between two voters

$$\text{conf}^{\circ}_{v,v'}(a,b) = \begin{cases} 0 & \textit{if } v(ab)\cdot v'(ab) > 0 \\ |v(ab)| \circ |v'(ba)| & \textit{otherwise} \end{cases}$$



Max Sum Conflict

$$\text{MaxSum}(P) = \underset{a,b\in C}{\text{argmax}} \sum_{v,v'\in V} \text{conf}^{+}(a,b)$$

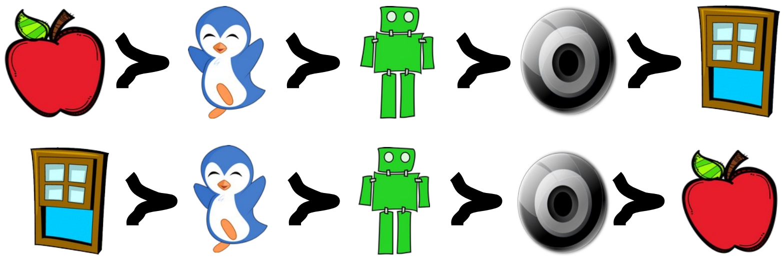
Max Nash Conflict

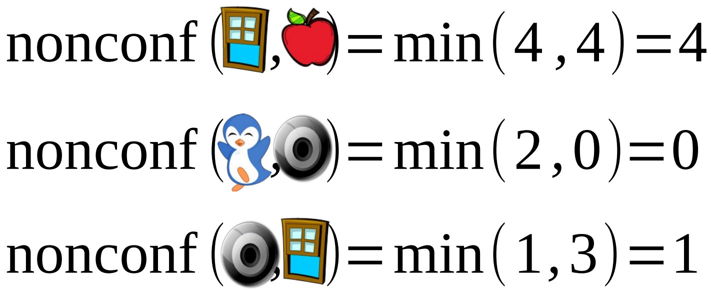
$$\text{MaxNash}(P) = \underset{a,b\in C}{\text{argmax}} \sum_{v,v'\in V} \text{conf}^{\times}(a,b)$$

Max Swap (intuitively)

Selects a pair, which requires the greatest number of swaps to make it non-conflicting.

$$\text{nonconf}(a,b)=\min\Big(\sum_{v\in V^{a\succ b}} v(ab),\ \sum_{v\in V^{b\succ a}} v(ba)\Big)$$





Max Swap

$$\text{MaxSwap}(P)=\underset{a,b\in C}{\text{argmax}}\ \text{nonconf}(a,b)$$

Understanding the ???

Understanding the conflictual voting rules

(axiomatically)

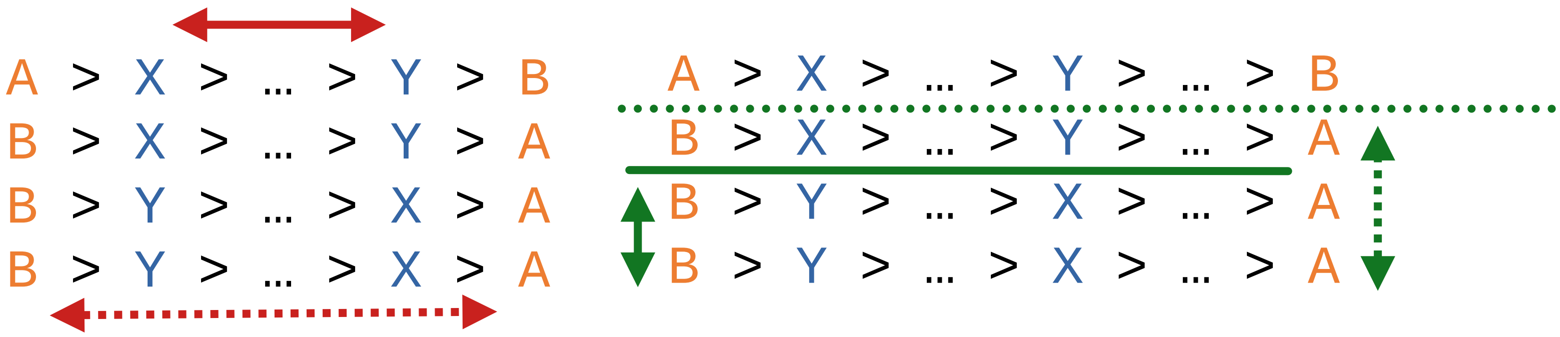
	MaxSum	MaxNash		MaxSwap
Reverse Stability	✓	✓		✓
Conflict Consistency	✓	✓		✓
Conflict Monotonicity	✗	✗		✗
Antagonization Consistency	✓	✓		✓
Matching Domination	✓	✓		✗

Axiomatic properties of conflictual rules.

Understanding the conflictual voting rules

(quantitatively)

Who is the most conflicting, {A, B} or {X,Y} ?



Discrepancy

$$\beta(a,b)=\frac{1}{n(m-1)}\sum_{v\in V} v(ab)$$

Partitioning ratio

$$\alpha(a,b)=\frac{2}{n}\min\left(|V^{a>b}|,|V^{b>a}|\right)$$

Max Sum $\mathrm{conf}^{+}(a,b)=C_2\,\alpha(2-\alpha)\,\beta$

Max Nash $\mathrm{conf}^{\times}(a,b)=C_1\,\alpha(2-\alpha)\,\beta^2$

Max Swap $\mathrm{nonconf}(a,b)=C_3\,\alpha\beta$

Max Polar

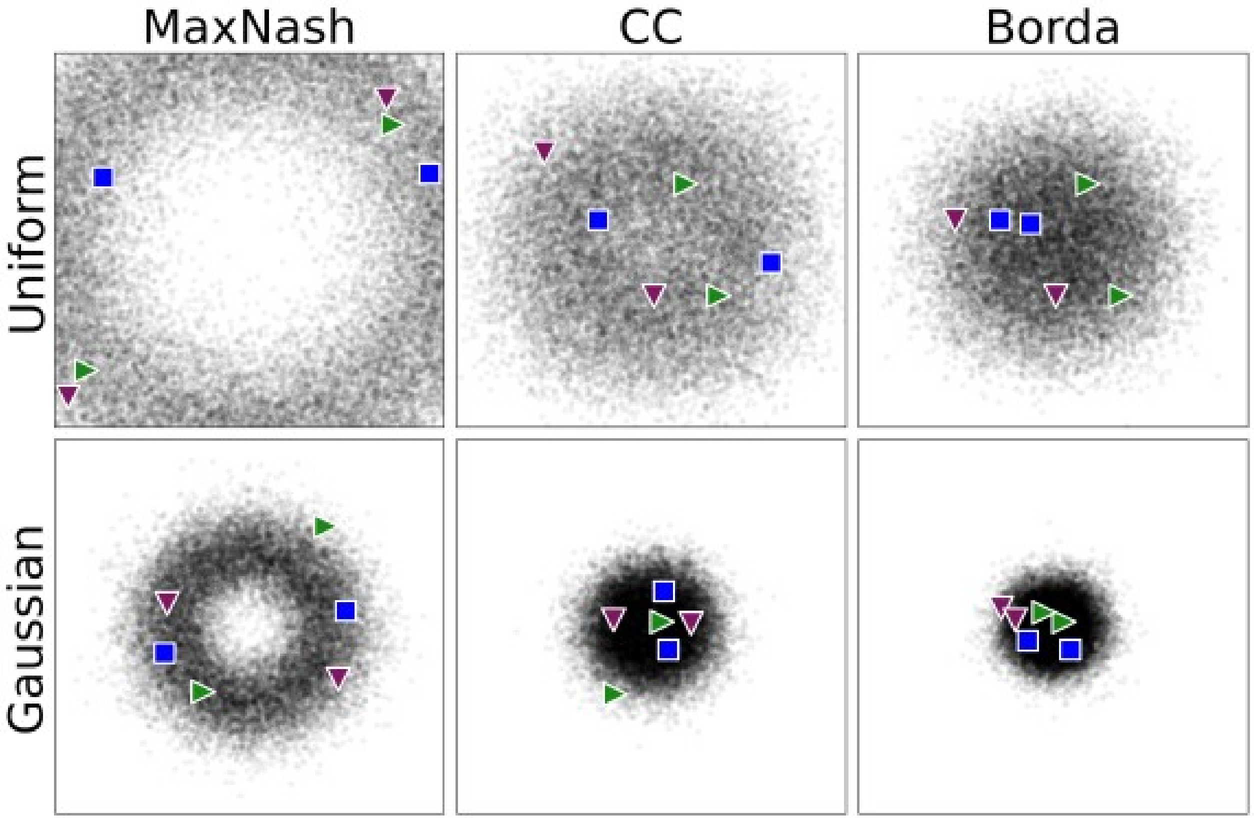
$$p\text{-}MaxPolar(P)=\operatorname*{argmax}_{a,b\in C}\alpha(a,b)\,\beta^p(a,b)$$

	MaxSum	MaxNash	MaxPolar	MaxSwap
Reverse Stability	✓	✓	✓	✓
Conflict Consistency	✓	✓	✓	✓
Conflict Monotonicity	✗	✗	✗	✗
Antagonization Consistency	✓	✓	✓	✓
Matching Domination	✓	✓	✓	✗

Axiomatic properties of conflictual rules.

Understanding the conflictual voting rules

(experimentally)

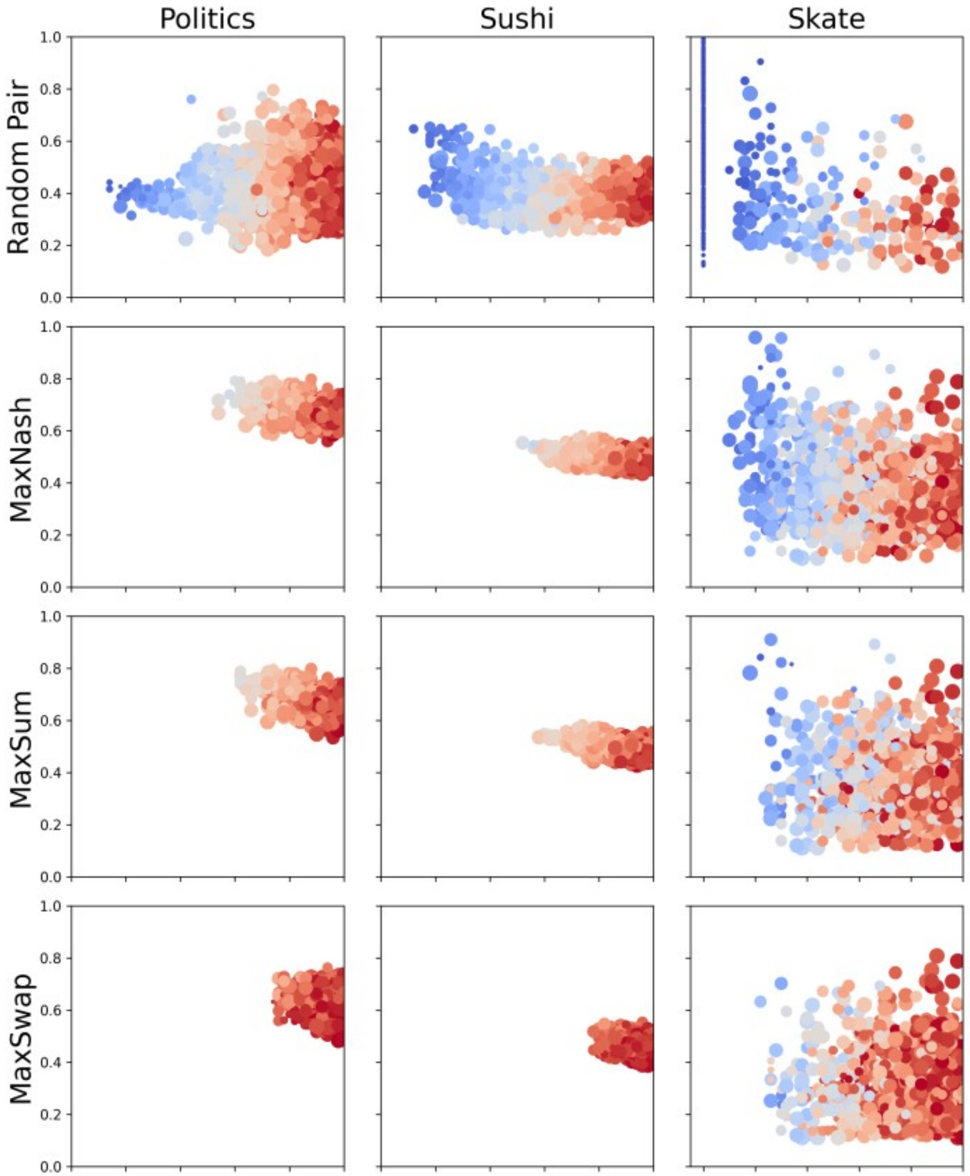

MaxNash
CC
Borda
Uniform
Gaussian

X axis: partitioning ratio α

the higher the more
balanced division of voters

Y axis: discrepancy β

the higher the more conflict
each pair generates



	2017			2022		
MaxSwap	Far-left	↔	Far-right	Far-left	↔	Far-right
MaxNash	Socialist	↔	Far-right	Left	↔	Far-right
MaxSum	Socialist	↔	Far-right	Far-left	↔	Far-right
2-MaxPolar	Far-left	↔	Far-right	Far-left	↔	Far-right
Borda	Left	↔	Liberal	Left	↔	Green
CC	Left	↔	Conservative	Green	↔	Far-right

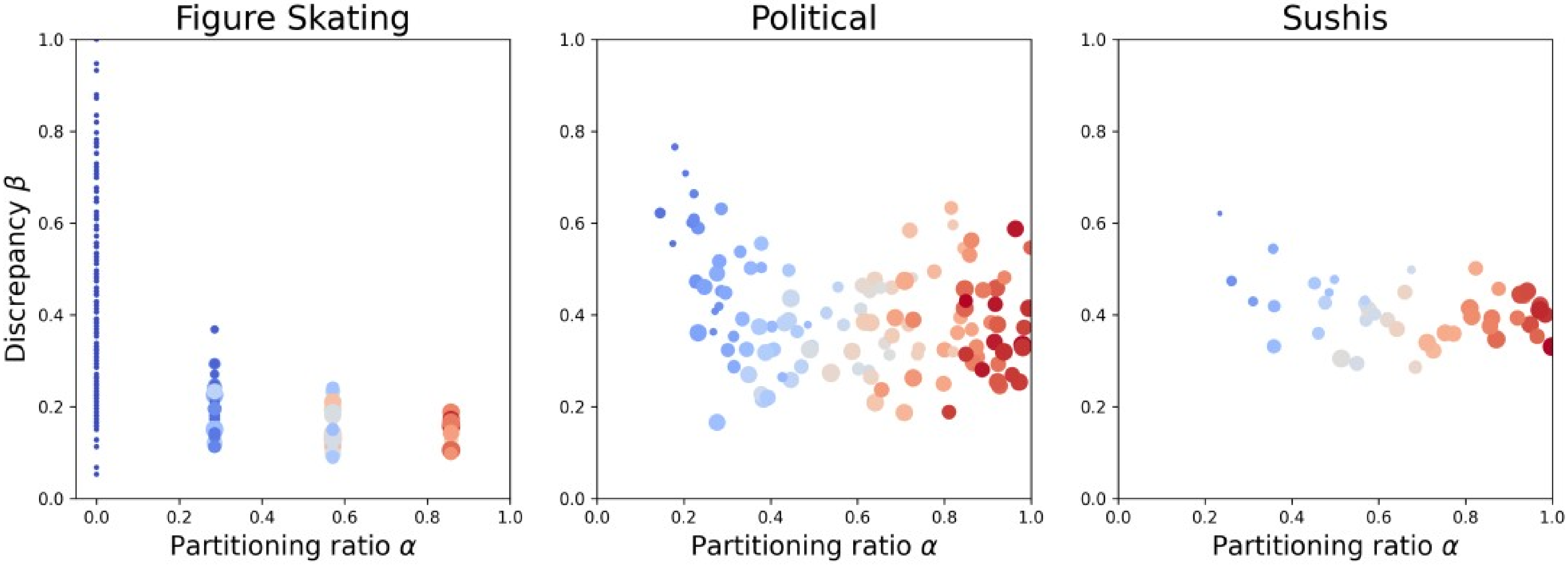


Figure Skating
Political
Sushis
Discrepancy β
Partitioning ratio α
Partitioning ratio α
Partitioning ratio α
0.0
0.2
0.4
0.6
0.8
1.0

recap

- Axioms
- Selection Rules
- Theoretical Validation
- Experimental Validation

More than two candidates?

Approval setting?

*Relation to full-election measures?

*Categorizing elections?

Thank you!